LBL-34928
UC-413

Lawrence Berkeley Laboratory
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Presented at the Perspectives in Nuclear Structure,
Copenhagen, Denmark, June 14–18, 1993,
and to be published in the Proceedings

Nuclear Physics: Macroscopic Aspects

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December 1993

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This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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Nuclear Physics: Macroscopic Aspects

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ABSTRACT

A systematic macroscopic, leptodermous approach to nuclear statics and dynamics is described, based formally on the assumptions $\hbar \to 0$ and $b/R << 1$, where b is the surface diffuseness and R the nuclear radius. The resulting static model of shell-corrected nuclear binding energies and deformabilities is accurate to better than 1 part in a thousand and yields a firm determination of the principal properties of the nuclear fluid. As regards dynamics, the above approach suggests that nuclear shape evolutions will often be dominated by dissipation, but quantitative comparisons with experimental data are more difficult than in the case of statics. In its simplest liquid drop version the model exhibits interesting formal connections to the classic astronomical problem of rotating gravitating masses.

1. INTRODUCTION

The earth, as seen from a spaceship, may be described approximately as a sphere. To apply this approximation to the description of the Alps would be foolish nonsense. Yet the approximation is useful in the proper context. Using a macroscopic approximation in nuclear physics is a little like saying that the earth is a sphere. Thus, to pretend that a nucleus is like a macroscopic droplet of nuclear matter is only useful if you stand back far enough to be willing to disregard shell effects and the quark-gluon structure of the nucleons themselves. In a sense that means disregarding the most interesting aspects of nuclear physics. Still, for some purposes it is useful to think of the earth as a sphere, and of a nucleus as a droplet of nuclear matter.

2. THE LEPTODERMOUS IDEALIZATION

What is the formal approximation according to which a nucleus becomes a droplet of nuclear matter? With some qualifications the answer may be stated as the *leptodermous idealization* [1]. This states that the thickness b of the nuclear surface is small compared to the nuclear radius R. More precisely, that the

presence of the surface is felt only in a region of limited thickness of order b. The surface thickness is determined by the range a of nuclear forces, which is of the order of the interparticle spacing, i.e., of the radius constant r_0. The radius constant is itself of the order of the Fermi wavelength $\lambda\!\!\!^{-}_F$ of the most energetic nucleon in the nucleus since, for a Fermi gas, $\lambda\!\!\!^{-}_F = (8/9\pi)^{1/3} r_0$. Thus the formal small expansion parameter in a leptodermous treatment is the dimensionless ratio ε, where

$$\varepsilon = b/R \propto a/R \propto r_0/R \propto \lambda\!\!\!^{-}_F/R \ . \tag{1}$$

The smallness of r_0/R suggests an expansion of nuclear properties in powers of $A^{-1/3}$, since $R = r_0 A^{1/3}$. The smallness of $\lambda\!\!\!^{-}_F/R$, if understood as implying formally that $\hbar \to 0$ (rather than that $R \to \infty$), leads to a semi-classical approach, such as the Thomas-Fermi approximation, analogous to the Thomas-Fermi approximation in atomic problems. (This approximation actually goes beyond the leptodermous idealization, in that it is useful even if b/R is not small [2,3].)

As usual, when one commits oneself to using a certain approximation, two questions naturally arise:

1. What are the consequences?
2. What are the limitations?

The second is the more difficult one, since it involves estimating effects beyond the idealization in question. Let us then start with the first question.

3. STATICS

If a system is truly leptodermous, the deviations from bulk behavior are confined to a thin surface region and one expects an expansion in b/R to be useful. Indeed, one can then write down the following expansion for the static energy of the system [4]:

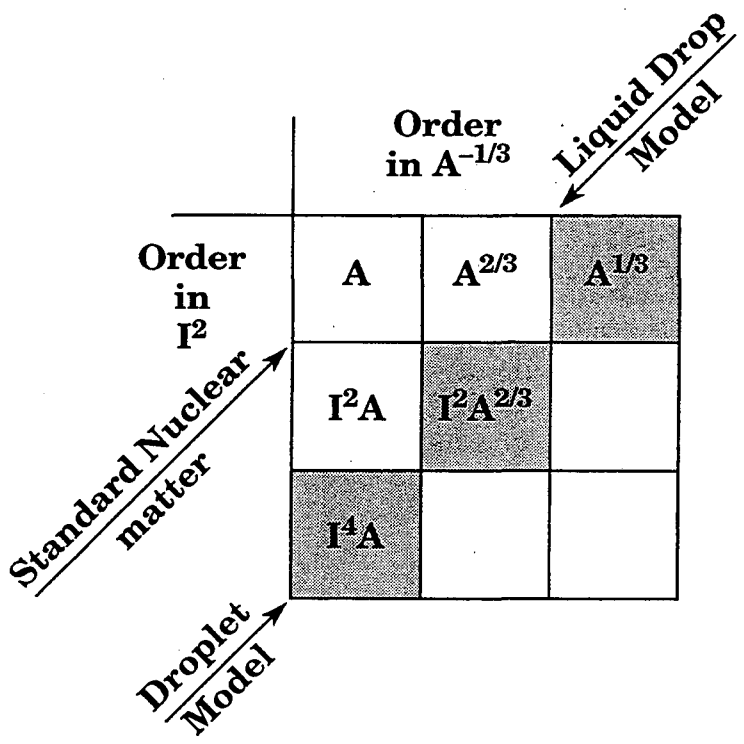
		Relative Order	
$E = c_1 \cdot \frac{4}{3}\pi R^3$	Volume Energy	1	A
$+c_2 \oint d\sigma$	Surface Energy	b/R	$A^{2/3}$
$+c_3 \oint \kappa d\sigma$	Curvature Energy	$(b/R)^2$	$A^{1/3}$
$+c_4 \oint \Gamma d\sigma$	Gaussian Curvature Energy	$(b/R)^3$	A^0
$+c_4' \oint \kappa^2 d\sigma$	Squared Curvature Energy	$(b/R)^3$	A^0

+ corrections that tend to zero as powers of $A^{-1/3}$

+ non-local terms non-analytic in b/R, e.g., of the form $e^{-R/b}$.

In the above the integrals are over the surface of the system, κ is the total curvature at a point on the surface (the sum of the reciprocals of the principal

radii of curvature, R_1 and R_2) and Γ is the Gaussian curvature $1/R_1R_2$. The quantities c_1, ... c_4' are constants independent of the system's size and shape (but dependent, in general, on the bulk density and composition of the nuclear droplet). For standard nuclear matter c_1 is proportional to the binding energy per particle and c_2 is the surface energy per unit area. If the binding energy per particle is taken to depend quadratically on the relative neutron excess according to the expression $-a_1 + JI^2$, where $I = (N-Z)/A$, if terms beyond the surface energy are neglected and if an electrostatic energy of a uniform charge distribution is added, one obtains the standard Liquid Drop model of nuclear masses and deformabilities. The model has four adjustable parameters: a_1, c_2, J and r_0. (Alternatively, one may impose on r_0 the value deduced from measurements of nuclear sizes.) If one wishes to go to the next order in $A^{-1/3}$ and I^2 *consistently*, i.e., in such a way that *all* terms of this order are included in the energy, one is led to the Droplet Model [5], based on the following hierarchy:



This self-consistency of the Droplet Model requires that the neutron and proton densities should deviate slightly from uniformity in the bulk *and* that they should be bounded by two slightly different effective surfaces. The latter leads to the appearance of a neutron skin. It turns out that the Droplet Model has now *nine* adjustable parameters, five more than the Liquid Drop model. The three most important of the new parameters are the compressibility coefficient K of nuclear matter, a coefficient a_3 (proportional to c_3) determining the curvature correction to the surface energy, and a coefficient Q, determining the effective resistance of the surface energy against the formation of a neutron skin. In a nuclear mass formula this coefficient is found to control the dependence of the surface energy term on the neutron excess (the so-called surface symmetry energy).

An even more ambitious macroscopic scheme is the Thomas-Fermi model [2,3,6,7,8], which not only goes beyond the Droplet Model, but does so with fewer parameters (typically six or seven). The price one pays for this is the loss of much of the algebraic convenience of the Liquid Drop or Droplet models.

What are such macroscopic models good for? There are three aspects:

1. The models are useful in a semi-empirical description of the binding energies of nuclei, of nuclear fission barriers (also at high spin) and, more generally, they provide the macroscopic part of the deformation energy in dynamical processes such as fission or nucleus-nucleus collisions.

2. The models are a tool for deducing various properties of the nuclear fluid (e.g., volume and surface energies) by fitting the models' adjustable parameters to experimental data.

3. Using the above information, the macroscopic models may then be used to estimate the equation of state of nuclear and neutron matter in astrophysical applications (neutron stars, supernovae explosions). This is where a macroscopic treatment is the only option available: you cannot use a microscopic Hartree-Fock theory to discuss 10^{57} nucleons in a neutron star.

Where do we stand today? Ground state binding energies are accounted for very well by macroscopic models, within the expected deviations due to shell effects. The principal properties of the nuclear fluid determined by fits to binding energies are [9]: $a_1 \approx 16.2$ MeV, $a_2 \equiv 4\pi r_0^2 c_2 \approx 24$ MeV, $J \approx 33$ MeV. When shell corrections calculated according to the Strutinsky method are allowed for, the RMS deviation between experimental and theoretical binding energies for some 1650 nuclei is about 0.67 MeV. A good part of this deviation is actually due to the limited accuracy of the Strutinsky estimate of shell effects for light nuclei. If the deviations for nuclei with $N < 65$ are left out, the RMS deviation in the remaining region is only about 0.45 MeV [9]. This is 0.45 MeV out of a total binding energy of some 1000 MeV for a medium heavy nucleus!

It is interesting that one can do almost as well with the Liquid Drop model, without the Droplet Model refinements. But not quite. For some time there has been evidence for a surface symmetry energy describable in the Droplet Model using a value of the Q parameter equal to about 30 MeV (with considerable uncertainty as to the precise value). More recently, evidence has also emerged in fits to binding energies for a finite value of the compressibility coefficient K. Figure 1, taken from Ref. 9, shows how the fit to ground state masses is improved if one goes from a Liquid Drop model (with $K = \infty$) to a Droplet Model with $K = 240$ MeV. (See also Ref. 10.) As regards the third Droplet Model refinement associated with the curvature correction to the surface energy, there is an interesting puzzle [11]. Fits to binding energies are happiest *without* a curvature correction term proportional to $A^{1/3}$, whereas various theoretical estimates suggest $a_3 A^{1/3}$, with $a_3 \approx 10$ MeV. How serious this might be is brought out by comparing calculated and measured fission barriers. (Because a deformed fission saddle point shape has an integrated curvature considerably different from that for a sphere, the curvature energy becomes relatively important.) Thus a recent refined Thomas-Fermi model, fitted to nuclear ground state masses and sizes, and which is characterized by a curvature correction coefficient $a_3 = 11$ MeV, when applied to fission, gives for ^{194}Hg a shell-corrected barrier of 25 MeV where

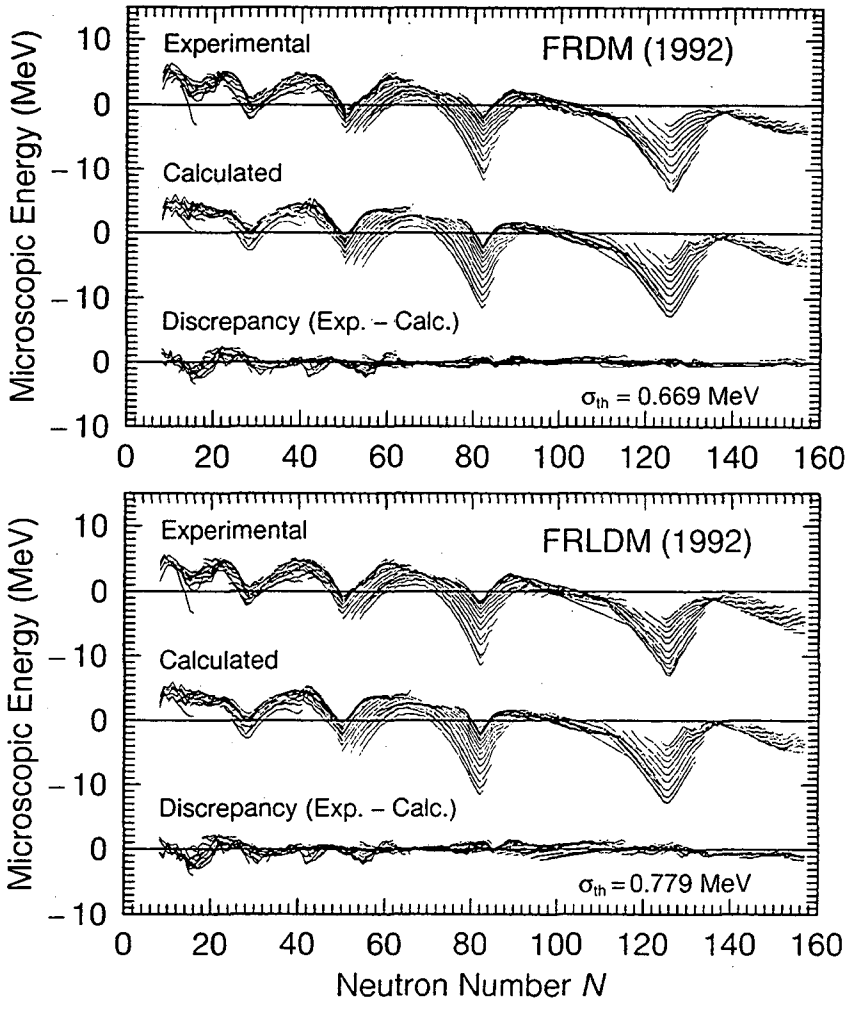


Figure 1. The top band in each panel shows the difference between measured masses of nuclei and droplet model masses (upper panel) and liquid drop model masses (lower panel). Lines connect isotopes of a given element. The middle bands show the calculated Strutinsky shell corrections. The bottom bands show the discrepancies between measured masses and shell-corrected droplet or liquid drop masses. For lighter nuclei (with $N \lesssim 64$) the discrepancy is due mostly to the limited accuracy of the shell correction. For heavier nuclei the improvement brought about by the droplet model is largely due to the finite value of the compressibility coefficient (K = 240 MeV). From Ref. 9.

measurements indicate 14 MeV [3]. By contrast, a model which (by construction) has no curvature correction (and does not insist on reproducing nuclear sizes) *can* reproduce the measured fission barriers (of some 28 nuclei) to within about 1 MeV [9]. Something is not understood here about the curvature correction and fission barriers.

A word about the still higher-order terms at the A^0 level in the leptodermous expansion. As a function of A they are constants. As a function of shape some of them have a peculiar behaviour that could make them important despite their relative smallness. Thus the term $\oint(1/R_1R_2)d\sigma$ is proportional to the Euler-

Poincaré topological invariant. It is strictly independent of shape, and changes only—but then suddenly—when the topology changes. Thus the above term is 4π for a single fragment of any shape, 8π for two fragments of any shape, zero for a torus, etc. This causes problems if such a term is kept in the binding energy formula, which is then used to describe fission. Thus, at the instant of scission, this term would jump discontinuously to twice its value! An open problem is how such a term *really* changes in the vicinity of scission, when the diffuseness of the nuclear surface is taken into account. There is then no well-defined scission point, but a fuzzed out scission neighborhood. We should ask mathematicians to work out for us a generalized Euler-Poincaré near-invariant for diffuse surfaces.

Strangely enough there is another term at the A^0 level with a similar unusual behavior. It is the so-called Wigner term, which is often included in binding energy formulae. It has the form $W|I|$, with $W \approx 30$ MeV. There is evidence for such a term in the measured masses of the lighter nuclei, and there are theoretical reasons to expect its presence [5]. The peculiar dependence on $|N-Z|$ is a reflection of the fact that the Wigner term has probably to do with the number of pairs of nucleons in identical orbits. [Think of a group of Z men and N women paired off as dancing partners. The number of couples is N or Z, whichever is less, and this can be written as $1/2(N+Z-|N-Z|)$]. The Wigner term is formally of order A^0 and, interestingly, the same dependence on shape seems to be implied as for the topological A^0 term: according to the simplest model [5] there should be no shape dependence until scission, followed by a sudden doubling. Since for a ^{264}Fm nucleus we have $I = 0.24$, the predicted jump at scission would be about 7 MeV. Again one needs a more careful analysis of how such a schematic jump is washed out in the case of real nuclei.

Finally a word about the non-analytic term $e^{-1/\varepsilon}$. Far from being an academic curiosity it is this type of term which is responsible for the so-called proximity interaction between the surfaces of two approaching nuclei, an interaction essential for the description of nucleus-nucleus collisions [4,12]. But even in the case of a *single* nucleus such a term is expected to be present. Most of it can be understood as resulting from a 'proximity' interaction of surface elements on the *opposite sides* of a nucleus. The presence of such a term is nicely illustrated by the (exact) formula for the interaction energy E of a prototype leptodermous system consisting of a uniform density ρ inside a sphere of radius R, whose elements interact via a Yukawa interaction of range a [13]:

$$E = a_1 A\left[-1+\frac{3}{2}(a/R)+0\cdot(a/R)^2-\frac{3}{2}(a/R)^3+\frac{3}{2}(a/R)\left(1+\frac{a}{R}\right)^2\exp(-2R/a)\right]. \quad (2)$$

Here a_1 is the appropriate volume energy coefficient and $A = (4/3)\pi R^3\rho$. In addition to the polynomial in (a/R) there appears a non-analytic term, exponential in the ratio of the sphere's diameter to the range of interaction. The retention of a term of this general type may be important in semi-empirical mass formulae, but the problem needs further study. (Note: the vanishing of the curvature correction term in Eq. (2) is not typical of more realistic models.)

4. A GLOBAL LOOK

Let us now forget all these higher order terms and go back to the incompressible liquid drop with simple surface and electrostatic energies, but generalized to incorporate a rotational energy calculated by assuming a common angular velocity for the drop's mass elements. There are two dimensionless parameters in this idealized gyrostatic problem. They specify the amount of charge and the amount of angular momentum on the drop. They may be chosen as the conventional fissility parameter x and the rotational parameter y, defined as follows:

$$x = \frac{\text{(electrostatic energy of spherical configuration)}}{2\ \text{(surface energy of sphere)}}$$

$$y = \frac{\text{(rotational energy of spherical configuration)}}{\text{(surface energy of sphere)}} \quad . \tag{3}$$

One can now pose the following grand problem [14,15]: given a pair of values (x,y), discuss the many-dimensional deformation energy landscape for such a drop; in particular determine all the stable and unstable configurations of equilibrium, i.e., minima, mountain tops, saddle-point passes with various degrees of instability, etc.; repeat this exercise for all values of x and y, positive and negative!

For x in the range 0 to 1 and small y, one is discussing idealized rotating nuclei in the periodic table. The results are useful in interpreting fission barriers of rotating nuclei and the existence of superdeformed spinning nuclei. See Fig. 2. But what does the rest of the x-y parameter space correspond to?

Negative x means that the repulsive electrostatic energy has been replaced by an attractive gravitational energy. For $x = -1/2$ the gravitational and surface energies are equal. For a globe of water this happens when the radius is about 10 m. So this regime of x-values would correspond, for example, to small asteroids when in a molten state. (The number of molecules in such an object is of the order of 10^{33}. You can readily verify by a dimensional argument that this is the order of the ratio of the electromagnetic to the gravitational coupling constant between molecules.) For still larger negative values of x the surface energy becomes negligible, and at $x \to -\infty$ we make contact with the classic 19th century problem of the equilibrium shapes of rotating, gravitating masses [16].

What about negative values of y? At first this sounds silly: a negative rotational energy or an imaginary angular momentum? Actually, negative y corresponds to nothing more exotic than bubbles in a uniformly rotating liquid. The mass of a bubble in relation to the surrounding medium is negative, and an air bubble in a sealed glass cylinder filled with water and rotating about its axis experiences a negative centrifugal force. With increasing angular velocity the bubble is drawn to the axis of rotation and assumes stretched-out *prolate* configurations of equilibrium. These configurations are a continuation to negative rotational energies of the well-known oblate Plateau shapes of rotating

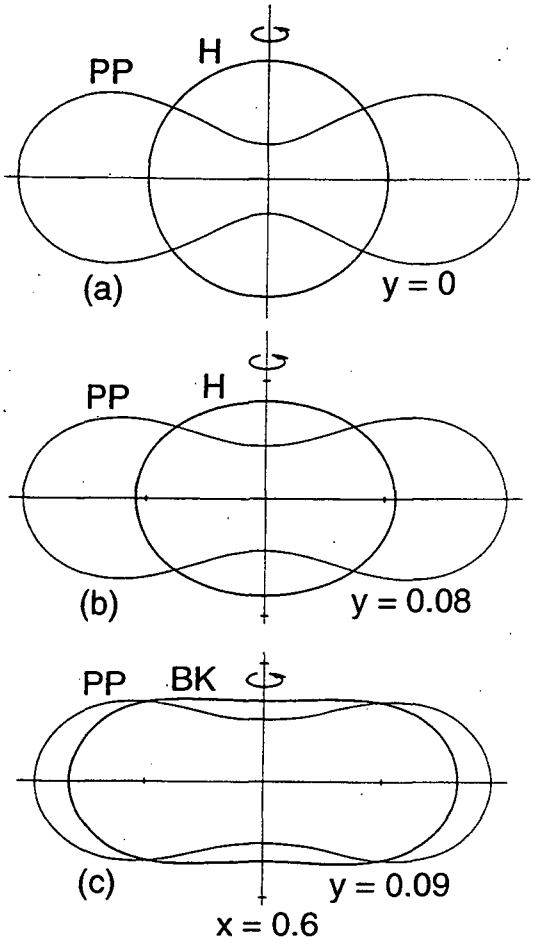


Figure 2. A pair of equilibrium shapes (ground state, labeled H, and saddle point, labeled PP) for a liquid drop with fissility parameter x = 0.6 and rotation parameter y = 0, 0.08, 0.09. The case y = 0.09 corresponds approximately to a superdeformed rare earth nucleus rotating about the vertical axis with an angular momentum of about $85\hbar$. In this case the two shapes shown have approximate rotational symmetry about the horizontal axis. From Ref. 14.

globes with surface tension (most conveniently studied in spaceships). When the bubble is in a gravitating or uniformly charged liquid the system corresponds to negative y and positive or negative x-values. (See Fig. 3)

Suppose we now calculate, for a given x,y, the set of equilibrium shapes that make the energy stationary. Suppose we label the n-th shape with some characteristic quantity, say its maximum extension L_n. Considered as functions of x and y these quantities $L_n(x,y)$ trace out sets of two-dimensional surfaces. If one of the parameters is frozen, for example if y is taken to be zero, we will have sets of curves depending on x. Figure 4 shows what some of these curves look like for $x > 0$. When the extra dimension y is added to the plot, the curves become families of surfaces that fold and cross in intricate ways. There are useful general rules which relate the degrees of instability of the equilibrium shapes that come together at the folds or crossings (Poincaré's rules of 'exchange of stabilities') [17].

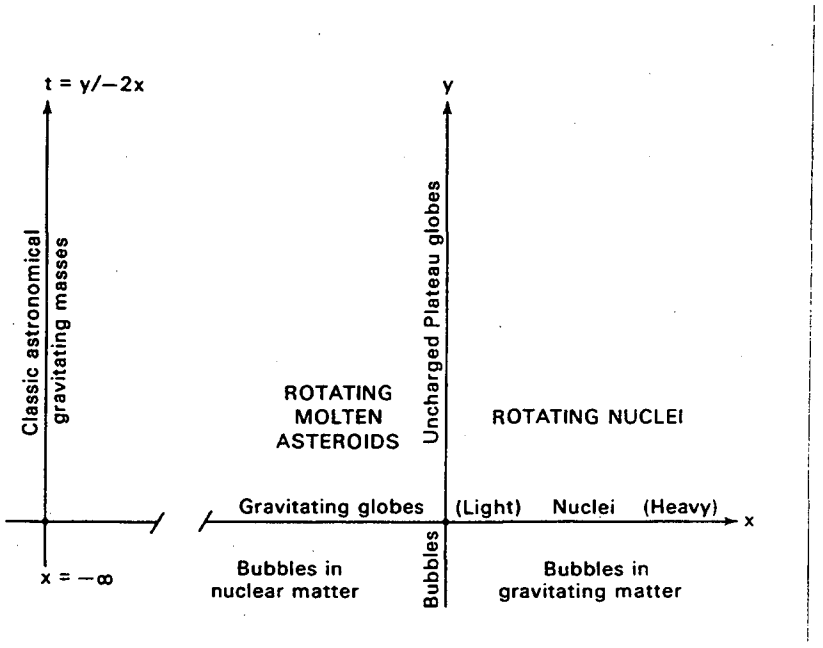


Figure 3. Physical systems corresponding to different regimes in the space of the fissility and rotational parameters x and y. From Ref. 15.

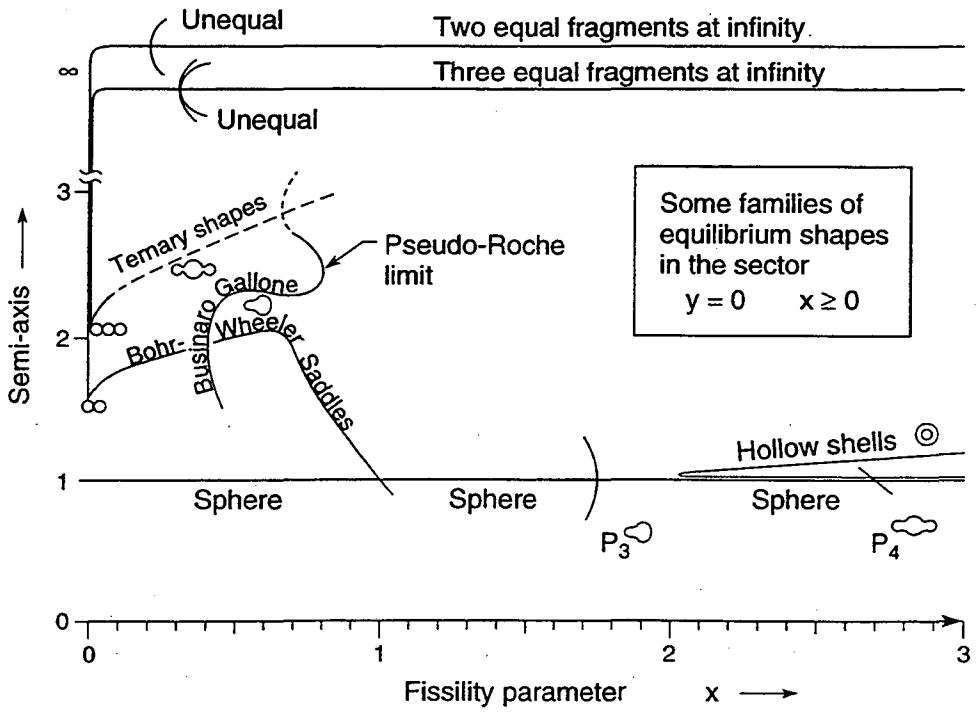


Figure 4. The connections between different families of equilibrium shapes can be illustrated by plotting some characteristic quantity (in this case one of the semi-axes) against one or more parameters specifying the system (in this case the fissility x, with y held fixed at zero). Figure based on Refs. 25–27 and unpublished work.

Imagine now that we project the locations of the folds and crossings onto the x-y plane. This divides the x,y parameter space into several domains with different physical meanings, as illustrated in Fig. 5. [The projections obey (for the most part) the canonical rules of "Catastrophe Theory" [18], but some generalization of the standard rules appears necessary.]

This somewhat abstract global way of generalizing the rotating nuclear liquid drop problem has, among other things, shed new light on the classic discussions of idealized astronomical masses studied through the centuries by Newton, Jacobi, Riemann, Poincaré, Darwin, Jeans, Lyapunov, Appell, and, more recently, by Chandrasekhar [16] and others. Figure 6 summarizes what was known until relatively recently about the locations of the most important families of astronomical equilibrium configurations, and indicates an attempt to relate them to each other and to a further set of "ghost families" required to avoid loose ends [15].

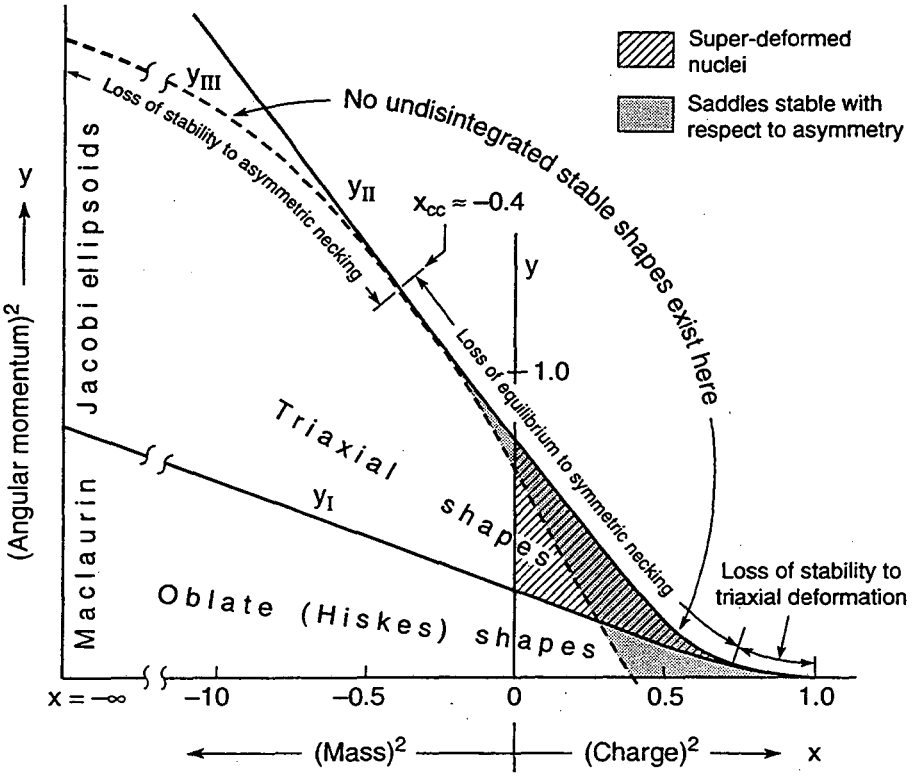


Figure 5. The regimes in the space of the fissility and rotational parameters x and y where the ground-state equilibrium shapes are oblate or triaxial. With increasing angular momentum these shapes disintegrate by loss of stability to a triaxial deformation if $0.73 \lesssim x < 1$, by loss of equilibrium to symmetric necking if $-0.4 \lesssim x \lesssim 0.73$ and by loss of stability to asymmetric necking if $-\infty \leq x \lesssim -0.4$. From Ref. 15 supplemented by Ref. 28.

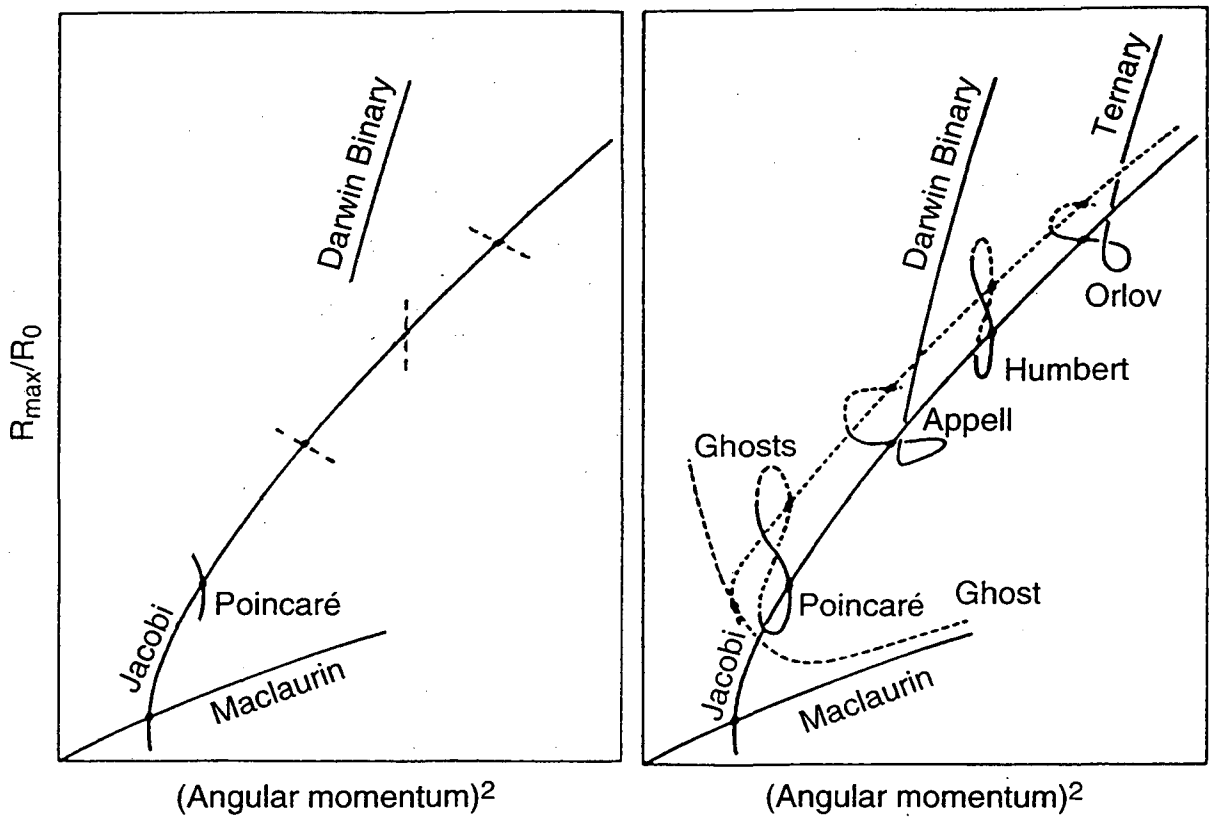


Figure 6. The major semi-axes R_{max} of rotating equilibrium shapes of uniform gravitating masses, plotted as a function of angular momentum squared. The right hand side shows how the conventional picture on the left may be completed by using insights gained from a global analysis that includes rotating idealized nuclei. A "ghost" is a conventional rotating configuration accompanied by a vanishingly small satellite in synchronous orbit. (Ref. 15.)

5. WHEN IS THE MACROSCOPIC APPROACH JUSTIFIED?

After this digression, back to the drop of nuclear matter and to the second, more difficult question: When is the macroscopic, leptodermous approach justified? The crucial approximations are $\hbar \to 0$ (no shell effects) and the localization of surface effects to a thin layer, $b << R$. For an ordinary liquid where both the range of inter-molecular forces and the molecular mean free paths are short, such a localization is well justified. But in the case of a nucleus the situation is more subtle. The range of the nuclear force is indeed small, but the mean free path is long rather than short. Does that pose a problem? It does indeed, if the nuclear mean-field potential is such that the nucleonic motions are integrable or nearly integrable. (A dynamical system is integrable if there are as many constants of motion as degrees of freedom. Examples: a particle in a rectangular box, a spherical or spheroidal box, a harmonic oscillator potential, isotropic or not.) In

such cases each particle has encoded in its behavior knowledge of the constants of motion that it has to respect. This means knowledge of a *global* property of the potential well. It is then unlikely that the properties of a fluid made up of such particles can be described by reference to localized surface conditions.

At the other extreme from integrable dynamics is chaotic dynamics, characterized by exponential sensitivity to initial conditions. In that case there is nothing special about any particular shape of the potential, and one shape is as good as another so long as it stays away from the subset of near-integrable configurations. In that case an averaged, leptodermous, macroscopic treatment might be relevant even when the mean free paths are long. These expectations are borne out by numerical studies of classical or quantal particles in various potential wells. As regards the *static* deformability of such systems the macroscopic-leptodermous approximation is found to be extremely good. Large deviations are indeed present for assemblies of particles whose dynamics is integrable, but even then the *average deformability* of the assembly is well described by the leptodermous expansion [1,19].

The net result is that we are now in possession of a semi-empirical description of the average static energy and deformability of a nuclear drop, based on the leptodermous expansion which, when corrected for shell effects, is accurate to better than an MeV — except near scission, where there are questions left unanswered.

6. DYNAMICS

Now we come to a new problem: can we say something equally simple about the *dynamic* properties of a nuclear drop, when the drop's shape is changing in time, as in fission or in nucleus-nucleus collisions? In particular, let me focus on the *dissipative resistance* to shape changes or, equivalently, on the rate of energy dissipation that would be expected when a nuclear drop is changing its shape at a given rate. Let us again make the following idealizations: macroscopic ($\hbar \to 0$), leptodermous ($b/R \ll 1$), together with the assumption of long mean free paths and chaotic nucleonic motions. Taken literally, these assumptions mean that we are dealing with a gas of independent point particles in the classical limit, moving chaotically in a slightly diffuse potential well. The well is now made to change its shape (at fixed volume) and the questions is: what is the dissipative resistance against this change or, equivalently, at what rate is the gas being heated up?

Since the potential well is *flat* in the bulk, the only transfer of energy from the wall motions to the particles *takes place in a thin surface region*. (This remains true even if the particles are quantized!) This immediately suggests that one should be able to write down the rate at which the gas is being heated up, dE/dt, as an expansion in b/R, analogous to the leptodermous expansion of E itself.

The result is the following dynamic analogue of the static leptodermous expansion [20,21]:

		Relative Order	
$\frac{dE}{dt} = k_1$ (volume integral) ≈ 0		—	—
$+k_2 \oint \dot{n}^2 d\sigma$	Wall formula	b/R	$A^{2/3}$
$+k_3 \oint \dot{n}^2 \kappa d\sigma$	Curvature correction	$(b/R)^2$	$A^{1/3}$
$+k_4 \oint \dot{n}^2 \Gamma d\sigma$			
$+k_4' \oint \dot{n}^2 \kappa^2 d\sigma$	Higher-order corrections	$(b/R)^3$	A^0
$+k_4'' \oint (\text{grad}\, \dot{n})^2 d\sigma$			

+ corrections that tend to zero as powers of $A^{-1/3}$

+ non-local (correlation) terms .

In the above, $\dot{n}$ specifies the normal speed of the deforming surface at the point in question, and grad $\dot{n}$ is the two-dimensional gradient of $\dot{n}$ considered as a function of position on the surface. The leading term in the leptodermous expansion is the "wall formula" for dissipation. The coefficient k_2 is the nuclear mass density ρ times the mean nucleonic speed v

$$k_2 = \rho v = (27/32\pi)(\pi/3)^{1/3}(\hbar / r_0^4) \approx 1.0 \times 10^{-22}\ \text{MeV sec fm}^{-4} . \tag{4}$$

Estimates of k_3 suggest $k_3/k_2 \approx 1$ fm. These values follow "from first principles" in the idealized model specified earlier. One could, however, regard the coefficients as (somewhat) adjustable parameters, in analogy with the semi-empirical approach to nuclear binding energies.

The numerical value of k_2 turns out to be such that in many cases nuclear dynamics would appear to be dominated by dissipation, i.e., inertial terms in the equation of motion should be negligible [22]. Let us then combine the lowest order (liquid drop) potential energy with the lowest order dissipation term (the wall formula) to obtain an equation of motion for the way the shape of an idealized nucleus would be expected to change with time. Imagine the displacement of the surface in time δt to be specified by δn. The energy dissipated is

$$\delta E = \frac{dE}{dt}\delta t = \rho v \oint \dot{n} \delta n d\sigma . \tag{5}$$

The change in the sum of electrostatic and surface energies is

$$\delta\ (\text{Potential Energy}) = \rho_e \oint \phi \delta n d\sigma + \gamma \oint \kappa \delta n d\sigma , \tag{6}$$

where ρ_e is the charge density, ϕ the electrostatic potential on the surface and $\gamma(\equiv c_2)$ is the surface energy per unit area. (I have made use of well-known expressions from electrostatics and analytical geometry of surfaces.) By conservation of energy the sum of Eqs. (5) and (6) should be zero for volume preserving deformations. This implies that

$$\rho v \dot{n} + \rho_e \phi + \gamma \kappa = \text{ constant.} \tag{7}$$

Taking the surface average of this equation determines the constant as $\rho_e \bar{\phi} + \gamma \bar{\kappa}$. (Bars denote surface averages. The surface average of $\dot{n}$ is zero by volume conservation.) There follows a delightfully simple equation of motion

$$\frac{dn}{dt} = P / \rho v \ , \tag{8}$$

where $P = \rho_e(\bar{\phi} - \phi) + (\bar{\kappa} - \kappa)$ is the pressure excess at a point on the surface due to the imbalance between electric and surface tension forces, and ρv is the constant given by Eq. (4).

This type of equation of motion, generalized for the presence of overall translations or 'drifts' and for the presence of necked-in shapes (when a 'window dissipation' appears), has been used in numerous studies of fission and nucleus-nucleus collisions [22,24]. Figure 7 shows the kinetic energy released in fission for nuclei ranging from medium to heavy. The calculation—without the adjustment of any parameters—reproduces the measurements fairly well. The upper curve in Fig. 7 shows what happens when the dissipation is switched off, and the lower curve the result of a calculation with a large viscosity of the conventional ('two-body') kind, appropriate for fluids consisting of particles with a mean free path short rather than long compared to the size of the system. By adjusting such a viscosity arbitrarily one could reproduce the measurements, but a short mean-free-path assumption is not appropriate at low nuclear temperatures. Figures 8 and 9 show other comparisons between the one-body (i.e., wall formula) dissipation and the two-body viscosity applied to the study of ternary fission.

The idealized dynamics represented by Eq. (8) (with refinements for drifts and constrictions) has also been applied to nucleus-nucleus collisions [23,24]. In particular, it has been used to study what happens when for two sufficiently heavy nuclei the Coulomb repulsion at contact is so large that the nuclei do not want to fuse and, after a period of amalgamation as a binary or 'composite' system, they reseparate. In such cases an extra push over and above the Coulomb barrier is required to form a compound nucleus. The strength of this extra push and the rate at which it grows with increasing size of the colliding nuclei should reflect the strength of the dissipative forces (which tend to reduce the efficacy of the extra collision energy in inducing fusion). Of the many comparisons between experiment and the macroscopic dynamic theory outlined above I have chosen Figs. 10 and 11. The former confirms the existence of the extra push phenomenon, but suggests that the idealized calculations, as they stand, predict a steeper increase of the extra push than is observed experimentally and that shell

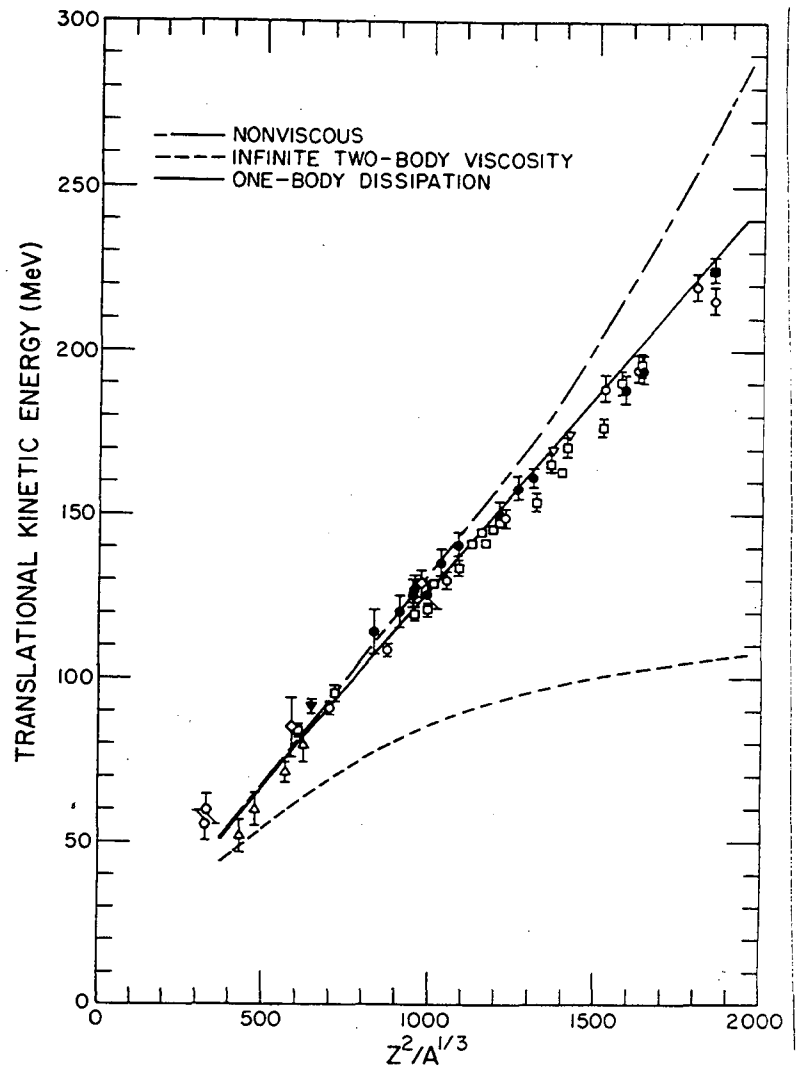


Figure 7. Fission fragment kinetic energies are compared with predictions of a liquid drop model incorporating a very large conventional (two-body) viscosity or one-body dissipation. (Ref. 22.)

effects play an important role. Figure 11, taken from the extensive comparisons of experimental data with the one-body dissipation theory in Ref. 24, suggests that the rate of mass transfer between two unequal nuclei in contact is roughly that predicted by theory. See also Ref. 23.

7. SUMMARY

I described an extreme macroscopic, leptodermous approach which considers the nucleus to be a droplet of a nuclear fluid. All structure effects are disregarded, or incorporated later as separate contributions. There are two aspects to the problem: static and dynamic. The static macroscopic model based on the leptodermous expansion with adjustable parameters is extremely

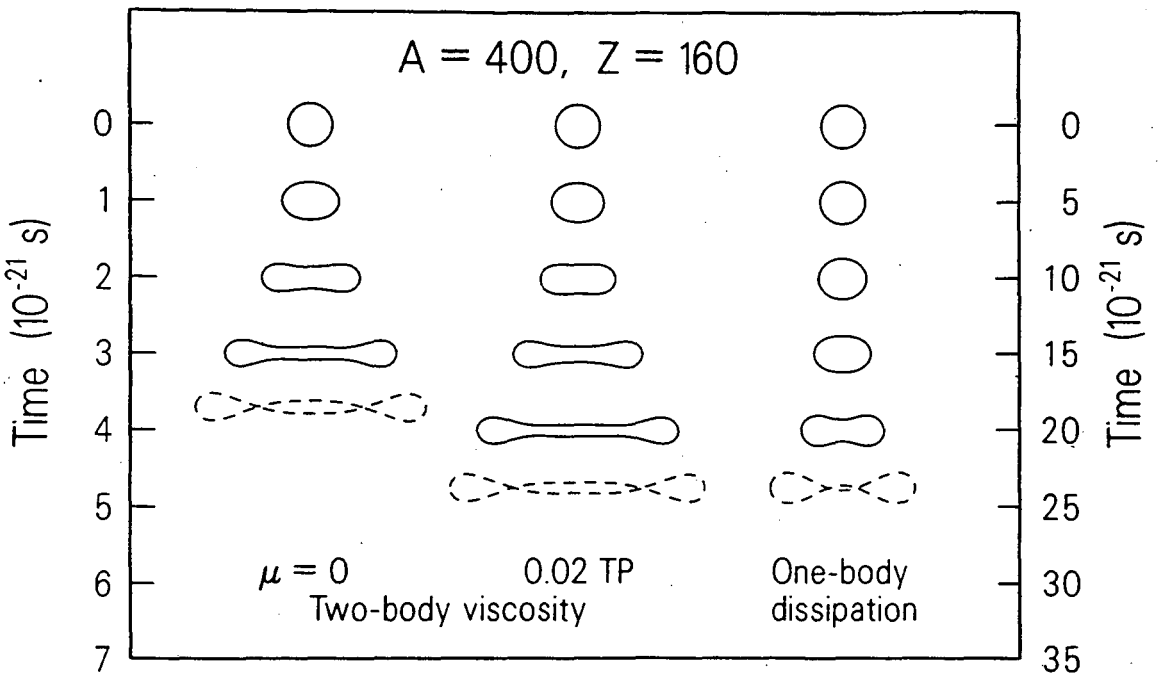


Figure 8. The difference between conventional (two-body) viscosity and one-body dissipation is illustrated by a liquid drop study of a hypothetical super-heavy nucleus. The conventional viscosity coefficient of 0.02 terapoise was adjusted to reproduce fission-fragment kinetic energies. Note the short time scale and the prediction of a well-developed third fragment at scission in the case of fission with conventional viscosity. (Ref. 29.)

successful. It describes shell-corrected nuclear deformation energies to within an MeV or better, except near scission, where there are problems. Using this model the major bulk and surface properties of the nuclear fluid have been determined, and estimates of secondary (droplet model) refinements have become available, except for the curvature correction coefficient, which remains a puzzle.

The dynamic macroscopic model makes an interesting qualitative prediction about nuclear dynamics being dominated by dissipation when the nucleonic motions are chaotic. When used in its extreme idealized form, without the adjustment of any parameters, the model is sometimes reasonably successful, but in one instance at least there is an indication that the unadorned wall formula may be overestimating the dissipation in nucleus-nucleus collisions.

In contrast to the statics, where the Strutinsky method has provided a good estimate of shell effects, the incorporation of quantal and shell effects into the macroscopic dynamics—in a simple way—has not progressed very far.

One might, in fact, say that the outstanding challenge is to develop a reasonably simple macroscopic-microscopic theory of nuclear dynamics analogous to the similar theory of nuclear statics.

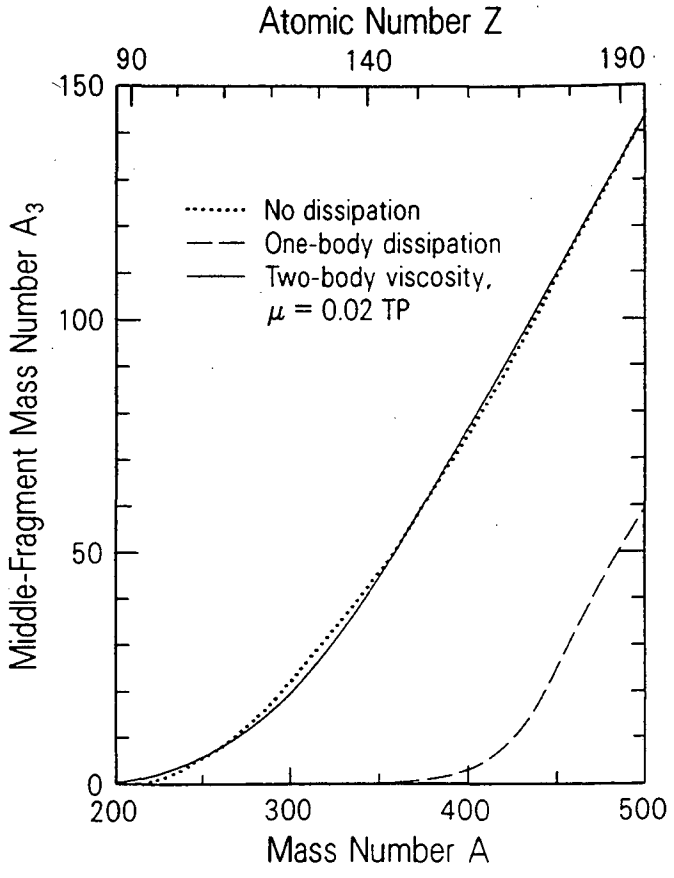


Figure 9. Using conventional viscosity one would predict that nuclei near the end of the periodic table would typically fission with a measurable amount of matter released between the main fragments at scission. With one-body dissipation ternary fission would not occur until much higher masses. (Ref. 29.)

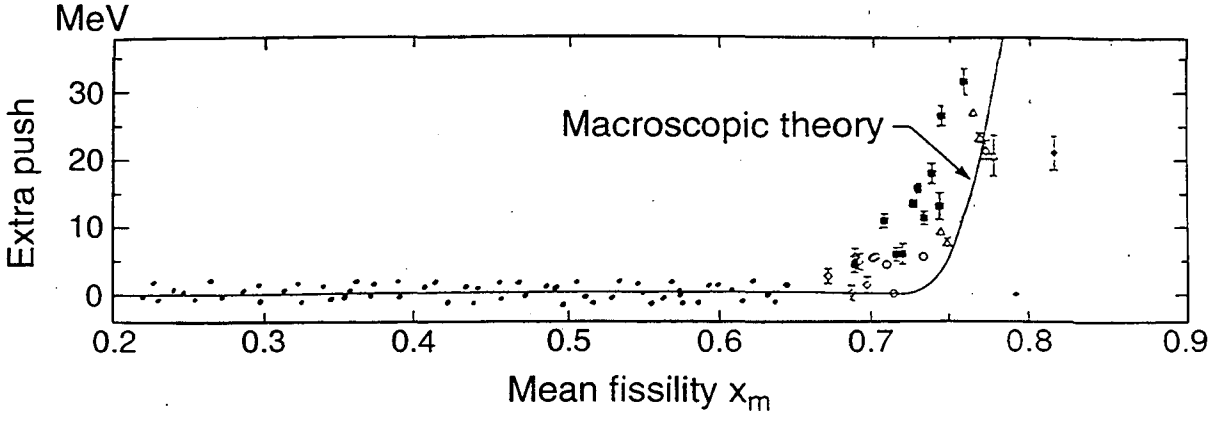


Figure 10. A schematic liquid drop model with one-body dissipation predicts that when a certain 'mean fissility' x_m exceeds a value near 0.72 a rapidly increasing 'extra push' energy in excess of the Coulomb barrier is needed to make the nuclei fuse. Experimental data appear to deviate significantly from this prediction. (For $x \lesssim 0.7$ measurements, shown schematically as dots, are consistent with zero extra push.)

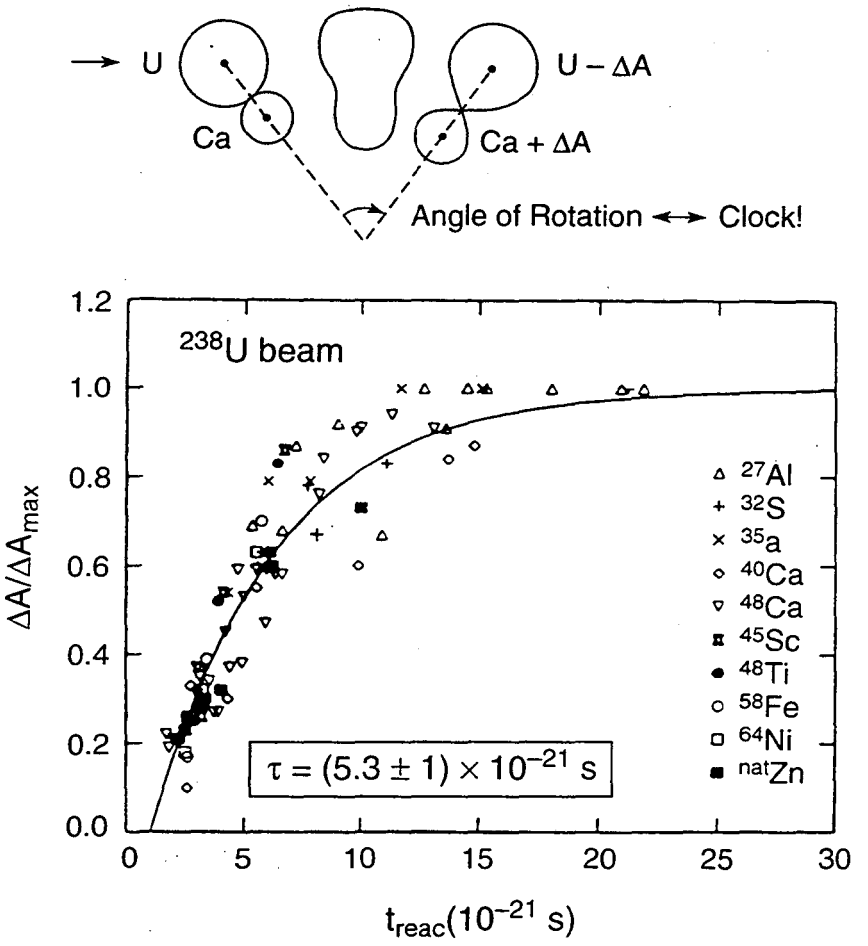


Figure 11. When the angle of rotation in a quasi-fission reaction is used as a clock to measure the rate at which mass flows from a U target into various projectiles, a time scale may be established for this type of asymmetric collective degree of freedom. The results are in semi-quantitative agreement with a liquid drop model incorporating one-body dissipation. (Ref. 24.)

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