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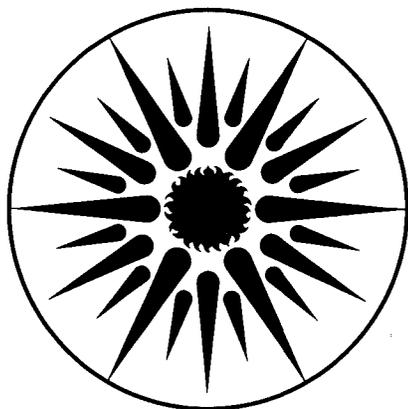
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J. Newman and W. Tiedemann

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John Newman

Department of Chemical Engineering  
University of California  
and  
Energy and Environment Division  
Lawrence Berkeley Laboratory  
University of California  
Berkeley, California 94720

and

William Tiedemann

Johnson Controls Battery Group, Incorporated  
Milwaukee, Wisconsin 53201

April 1994

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## Temperature Rise in a Battery Module

with Constant Heat Generation

John Newman<sup>a</sup> and William Tiedemann<sup>b</sup>

<sup>a</sup>Energy and Environment Division, Lawrence Berkeley Laboratory, and

Department of Chemical Engineering, University of California, Berkeley

<sup>b</sup>Johnson Controls Battery Group, Incorporated, Milwaukee, Wisconsin 53201

### Abstract

A three-dimensional battery module in the shape of a block is taken to generate heat uniformly throughout. The temperature rise as a function of time is worked out based on equations for heat conduction in solids. After appropriate nondimensionalization, the maximum temperature rise depends on the thermal aspect ratios (defined herein). By a superposition integral, the method can be extended to a time-dependent heat-generation rate, as appropriate for a driving profile.

### Introduction

In designing a thermal-management system, one should first establish an acceptable temperature range within which all battery components, *i.e.*, active material, binders, separator, electrolyte, *etc.*, should be maintained at all times. The temperature limits may be determined by life specifications, performance ratings, safety considerations, or avoidance of extremely low or high temperatures. Battery life and uniformity and predictability of performance are favored by a narrow range, while reducing system complexity, expense, and weight are favored by a somewhat wider range.

The battery pack should be placed in an insulated enclosure through which a fluid, such as air, at the minimum temperature is circulated. Design of a thermal-management system should therefore focus on minimizing the temperature variation among and within

modules of the battery pack. Of interest here could be lithium/polymer, high-temperature Li/FeS<sub>2</sub>, nickel/metal hydride, nickel/cadmium, or recombinant lead-acid systems.

Our attention here is focused on the temperature rise within the battery module placed in this constant-temperature environment. Whether the module is monopolar or bipolar, it is taken to be a layered structure, where separators and battery plates and partitions constitute the layers. Average thermal conductivities can be used, but different values will prevail in the directions parallel and perpendicular to the layers. An average heat generation rate  $q$  can also be used, because opposing positive and negative active material will be at nearly the same temperature. The module will be taken to be a parallelepiped, extending from 0 to  $L_x$  in the x-direction, and similarly in the y- and z-directions.

A much more complex level of analysis would be involved if it were necessary to account for the nonuniform current-density distribution across the face of a plate in a monopolar system<sup>1</sup> or the dependence of the heat-generation rate on the temperature at different points within the module (for example, because the electrical conductivity of the electrolyte changes). One-dimensional models have accounted for changes of heat-generation rates and physical properties with both temperature and state of charge, including the interaction of temperature and discharge characteristics, for many years.<sup>2,3,4</sup> It is useful for simpler design considerations to assert that the current and temperature distributions must be fairly uniform in a well-designed system and then work backwards through the electrical and thermal design to assure that they are.

Chen and Evans<sup>5</sup> show that the internal heat-transfer resistance can often predominate over the heat-transfer resistance at the periphery of the module. They also review some earlier work on thermal modeling of battery systems.

#### Problem Statement

The mathematical problem for the temperature distribution thus takes the form

$$\rho \hat{C}_p \frac{\partial T}{\partial t} = k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} + q. \quad (1)$$

The term on the left represents the accumulation of thermal energy per unit volume of the module; the density  $\rho$  and heat capacity per unit mass  $\widehat{C}_p$  would also represent averages over the materials in the layers. The first three terms on the right represent the net input into a unit volume by heat conduction in the three directions; the average thermal conductivities in these directions are denoted by  $k_x$ ,  $k_y$ , and  $k_z$ . The thermal generation rate  $q$  is the last term. The reader may wish to consult standard references on heat transfer<sup>6,7,8</sup> to clarify these concepts.

Let  $T_o$  be both the initial temperature and the lower temperature limit at which the boundaries of the module are maintained. The boundary and initial conditions are thus

$$\left. \begin{aligned} T = T_o \text{ at } x = 0, \quad T = T_o \text{ at } x = L_x, \\ T = T_o \text{ at } y = 0, \quad T = T_o \text{ at } y = L_y, \\ T = T_o \text{ at } z = 0, \quad T = T_o \text{ at } z = L_z, \end{aligned} \right\} \quad (2)$$

and

$$T = T_o \text{ at } t = 0. \quad (3)$$

All the parameters, including  $q$ , are taken to be uniform and independent of time.

We return later to the case of a heat-generation rate which depends on time. For economy in the development, we introduce the following dimensionless quantities:

$$\left. \begin{aligned} X = \frac{x}{L_x}, \quad Y = \frac{y}{L_y}, \quad Z = \frac{z}{L_z}, \\ \Theta = \frac{k_x(T - T_o)}{qL_x^2}, \quad \tau = \frac{k_x t}{\rho \widehat{C}_p L_x^2} \end{aligned} \right\} \quad (4)$$

Let the x-direction be that direction having the smallest value of  $L^2/k$ . Then we have made the temperature dimensionless with respect to the direction with the greatest heat-transfer rate. The dimensionless problem now becomes

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial X^2} + r_y^2 \frac{\partial^2 \Theta}{\partial Y^2} + r_z^2 \frac{\partial^2 \Theta}{\partial Z^2} + 1 \quad (5)$$

with the boundary and initial conditions

$$\left. \begin{aligned} \Theta = 0 \text{ at } X = 0, \quad \Theta = 0 \text{ at } X = 1, \\ \Theta = 0 \text{ at } Y = 0, \quad \Theta = 0 \text{ at } Y = 1, \\ \Theta = 0 \text{ at } Z = 0, \quad \Theta = 0 \text{ at } Z = 1, \end{aligned} \right\} \quad (6)$$

and

$$\Theta = 0 \text{ at } \tau = 0. \quad (7)$$

The *thermal aspect ratios* appearing in equation 5 are defined as

$$r_y = \frac{L_x}{L_y} \left( \frac{k_y}{k_x} \right)^{1/2} \quad \text{and} \quad r_z = \frac{L_x}{L_z} \left( \frac{k_z}{k_x} \right)^{1/2}. \quad (8)$$

These ratios emphasize that the relative importance of heat transfer in different directions is determined by a combination of the length and the thermal conductivities in these directions. It is clear that only these two parameters remain in the problem. By the choice of the x-direction as that direction having the smallest value of  $L^2/k$ , the thermal aspect ratios are less than or equal to 1, and the x-direction is dominant in determining the maximum temperature. The method of solving this problem is discussed further in the Appendix.

### Results and Discussion

Our design approach seeks to have the battery system operate below a given temperature maximum. We thus focus attention on the location of the highest temperature in the module, at the center ( $X = Y = Z = 0.5$ ). For presentation of the results, the dimensionless temperature and time are modified to include the thermal aspect ratios in the three directions. This also partially accounts for the possibility of heat being conducted in three directions. Thus, we define

$$T' = 8\Theta(1 + r_y^2 + r_z^2) = 8 \frac{T - T_o}{q} \left( \frac{k_x}{L_x^2} + \frac{k_y}{L_y^2} + \frac{k_z}{L_z^2} \right) \quad (9)$$

and

$$t' = \tau(1 + r_y^2 + r_z^2) = \frac{t}{\rho \bar{C}_p} \left( \frac{k_x}{L_x^2} + \frac{k_y}{L_y^2} + \frac{k_z}{L_z^2} \right). \quad (10)$$

Figure 1 presents the maximum value of this dimensionless temperature at the center of the module as a function of the dimensionless time for several combinations of the thermal aspect ratios. The adiabatic temperature rise comes from the first and last terms in equation 5; this gives a slope of  $\partial\Theta / \partial\tau=1$ , and the corresponding slope is 8 on Figure 1 for the modified  $t'$  and  $T'$  variables. All the curves have this slope at zero time, independent of the thermal aspect ratios. The curves then bend over and approach steady-state values  $T'_{ss}$ , which depend on the thermal aspect ratios.

As a first step in correlating the behavior in Figure 1, we show in Figure 2 how  $T'_{ss}$  depends on the aspect ratios. The values are plotted against  $r_y^2(2-r_y^2)+r_z^2(2-r_z^2)$ , with  $r_z/r_y$  as a parameter.

With a plot like Figure 2 for  $T'_{ss}$ , we can propose a formula for the transient curves on Figure 1. One possibility is

$$T' = T'_{ss} - Ce^{-\pi^2 t'} + (C - T'_{ss})e^{-Bt'}, \quad (11)$$

where  $B = (\pi^2 C - 8) / (C - T'_{ss})$  in order to give the correct slope at  $t'=0$  and  $C$  can be given a value like 1.5. Good values of  $B$  and  $C$  to use for certain combinations of  $r_y$  and  $r_z$  are given in Table 1. For the cube, and in general, the most slowly decaying transient term of the series for  $T'$  at the center is  $-(512 / \pi^5)\exp(-\pi^2 t')$ , and this is the basis for  $C$  in the table. With the value of  $T'_{ss}$  (see Figure 2), this gives  $B$ . As one of the aspect ratios approaches zero, more transient terms have exponents close to  $-\pi^2 t'$ , and when, say,  $r_z$  exactly equals zero, the most slowly decaying transient term becomes instead  $-(128 / \pi^4)\exp(-\pi^2 t')$ . Here the problem becomes two dimensional instead of three dimensional. Finally, as both thermal aspect ratios approach zero, still more transient terms have similar exponents. When  $r_y$  and  $r_z$  are both identically zero, the most slowly decaying transient term becomes  $-(32 / \pi^3)\exp(-\pi^2 t')$ . As the value of  $T'_{ss}$  drops on Figure 2, one may want to use successively smaller values of  $C$ , according to the above guidelines. Even without the best value of  $C$ , Equation 11 can be sufficiently accurate because it reproduces the correct initial slope and steady-state value.

Table 1. Appropriate values of  $B$  and  $C$  to use for a one-dimensional slab, a square column, and a cube (bottom to top in the table). Values of  $B$  should be calculated from the value of  $T'_{ss}$ .

$r_y$	$r_z$	$C$	$B$
1	1	1.673	26.28
1	0	1.314	36.73
0	0	1.032	68.20

We shall use as an example a lithium/polymer dual ion-insertion system<sup>9</sup> with a lower limit of 90°C and an upper limit of 110°C. This cell has four layers with the properties given in Table 2. The capacity of the cell is 19.22 C/cm<sup>2</sup>, and the average open-circuit potential is 3.5 V. The internal resistance is taken to be 50 Ω·cm<sup>2</sup>. For a 3-hr discharge, one can calculate the current density and hence a heat-generation rate  $q = 0.0031$  W/cm<sup>3</sup>.

Next we can specify the battery size to be 40 kW·hr. This gives a battery volume of  $1.13 \times 10^5$  cm<sup>3</sup>; if we make this into a cube, it will be 48.3 cm on a side. Thermal aspect ratios would be  $r_y = 1$  (since  $k_{||}$  applies in two directions) and  $r_z = 0.241$ . From Figure 2, we can take  $T'_{ss}$  to be 1.21, and  $C$  is taken to be 1.45. The steady-state temperature rise would thus be estimated to be 3.27 K, and the value of  $t'$  at 3 hours is 0.93. Since this is effectively a large value (see Figure 1), the estimated temperature rise is very close to the steady-state value. The adiabatic temperature rise under these conditions is calculated to be 20.09 K. This emphasizes the importance of heat conduction from the module.

Table 2. Thermal properties of the layers of a dual ion-insertion cell.

	thickness	average density	average heat capacity	average thermal conductivity
	$\mu\text{m}$	$\text{g}/\text{cm}^3$	$\text{J}/\text{g}\cdot\text{K}$	$\text{W}/\text{m}\cdot\text{K}$
negative electrode	243	1.6	0.838	1.17
separator	50	1.2	1.46	0.16
positive electrode	200	2.81	0.713	8.23
bipolar partition	20	5.83	0.318	317.5
cell	513	2.20	0.755	
perpendicular				0.941
parallel				16.2

The calculated temperature rise is well within the 20-K limit set initially. Otherwise, it would have been necessary to reconfigure the battery—perhaps by breaking it up into smaller modules or by changing the aspect ratio—to attain the design specification.

With the figures and the methodology, it is easy to estimate that the limit on temperature rise would be encountered if the time of discharge were shortened to 1.23 hr. In this case, the battery size changes because the average discharge potential is smaller. Inclusion of the thermal resistance of the wall of the module would also increase the estimated temperature rise.

If a battery is taken through a driving cycle, instead of being discharged at a constant current, the heat-generation rate and temperature rise will be greater.<sup>10</sup> With an ohmic model, such as that used here, the heat-generation rate can be higher by a factor of

$$F = t_d \int_0^{t_d} i^2 dt / \left[ \int_0^{t_d} i dt \right]^2 \quad (12)$$

Table 3 illustrates two possible driving profiles, one with regenerative braking and one without. The factor  $F$  for these profiles work out to be 3.69 and 2.4, respectively. With larger values of the factor, the temperature limits of the system are appropriately more stringent. With a factor of  $F = 3.69$ , the temperature rise in the above example becomes 12.09 K, and the adiabatic temperature rise is 74 K.

Table 3. Relative current densities for acceleration, cruise, braking, and rest for driving profiles with and without regenerative braking.

duration s	relative current density	
	profile 1	profile 2
25	4	4
50	1	1
15	-1.6	0
30	0	0

It would be useful to use a more detailed model<sup>9</sup> including diffusion limitations to verify that the proposed discharge time, attainable capacity, and average discharge potential are reasonably estimated. Such a model may also provide a more accurate estimate of the heat-generation rate.

*Time dependence of  $q$ .*—Carslaw and Jaeger suggest that a superposition integral can be used when the rate of heat generation depends on time. The previous development can thus be extrapolated easily to the time-dependent case. Take  $T'(t')$  to be the solution just obtained. (This can be at any given position in the module, and we suppress the dependence on  $X$ ,  $Y$ , and  $Z$ . However, the focus continues to be on the center of the

module, where we are most concerned about temperature extremes.) If a step change in  $q$  occurs at  $t' = t'_0$ , then the contribution to the temperature inside the module at a later time  $t'$  will be proportional to this step change and to  $T'(t' - t'_0)$ . The superposition integral takes the form

$$T = T_o + \frac{1}{8 \left( \frac{k_x}{L_x^2} + \frac{k_y}{L_y^2} + \frac{k_z}{L_z^2} \right)} \int_0^{t'} \frac{dq}{dt'} \Big|_{t'=t'_0} T'(t' - t'_0) dt'_0. \quad (13)$$

This equation could be applied to batteries run with a driving profile, including significant time parked (where one has to say how the coolant temperature will be maintained at the minimum limit) and time charging. Weekly temperature profiles could even be developed.

#### Appendix

The solution to equation 5 and the accompanying conditions can be written as

$$\Theta = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} A_{k,l,m} \left( 1 - e^{-\lambda_{k,l,m}^2 \tau} \right) \sin(\pi k X) \sin(\pi l Y) \sin(\pi m Z), \quad (A.1)$$

a form obtained by the method of separation of variables. This form already satisfies the boundary and initial conditions. Substitution into equation 5 gives the values of the eigenvalues:

$$\lambda_{k,l,m}^2 = \pi^2 \left( k^2 + r_y^2 l^2 + r_z^2 m^2 \right) \quad (A-2)$$

(in order to cancel the time-dependent terms) and a Fourier series describing the steady-state part:

$$1 = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} A_{k,l,m} \lambda_{k,l,m}^2 \sin(\pi k X) \sin(\pi l Y) \sin(\pi m Z). \quad (A.3)$$

The orthogonal property of the sine functions allows the coefficients to be determined. Multiply equation A.3 by  $\sin(\pi K X)$  and integrate with respect to  $X$  from 0 to 1. The terms for  $k \neq K$  drop out. Carry out similar operations for the y- and z-directions. For **odd** values of  $k$ ,  $l$ , and  $m$ , the result is

$$A_{k,l,m} = \frac{64 / \pi^5 klm}{k^2 + r_y^2 l^2 + r_z^2 m^2}. \quad (\text{A.4})$$

By the symmetry of the system,  $A_{k,l,m}$  is zero if any one of  $k$ ,  $l$ , and  $m$  is even.

Unfortunately, when the terms are evaluated at the center of the module, the signs will alternate. Thus, the steady-state value at the center must be evaluated from the form

$$T'_{ss} = \frac{512}{\pi^5} \sum_{k,l,m \text{ odd}}^{\infty} \frac{(1 + r_y^2 + r_z^2)(-1)^{(k+l+m-3)/2}}{klm(k^2 + r_y^2 l^2 + r_z^2 m^2)}. \quad (\text{A.5})$$

Simpler expressions can be developed if  $r_y$  or  $r_z$  or both are zero because one or more directions can then be dropped before attacking the problem. Such cases also apply if two opposite sides of the module are insulated and heat must be conducted out through the remaining sides.

#### Acknowledgment

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#### References

1. William H. Tiedemann and John Newman, "Current and Potential Distribution in Lead-Acid Battery Plates," Sidney Gross, ed., *Proceedings of the Symposium on Battery Design and Optimization, 79-1*, 39-49. Princeton, N. J.:The Electrochemical Society, Inc., 1979.
2. William H. Tiedemann and John Newman, "Mathematical Modeling of the Lead-Acid Cell," Sidney Gross, ed., *Proceedings of the Symposium on Battery Design and Optimization, 79-1*, 23-38. Princeton, N. J.:The Electrochemical Society, Inc., 1979.
3. Richard Pollard and John Newman, "Mathematical Modeling of the Lithium-Aluminum, Iron Sulfide Battery. I. Galvanostatic Discharge Behavior," *J. Electrochem. Soc.*, 128 (1981), 491-502.

4. Carolyn R. Pals, Marc Doyle, Thomas F. Fuller, and John Newman, "Modeling of Adiabatic and Isothermal Galvanostatic Discharge Behavior of the Lithium/Polymer/Insertion Cell," The Electrochemical Society, extended abstract for San Francisco, May, 1994.
5. Yufei Chen and James W. Evans, "Heat Transfer Phenomena in Lithium/Polymer-Electrolyte Batteries for Electric Vehicle Application," *J. Electrochem. Soc.*, 140 (1993), 1833-1838.
6. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids* (Oxford: Clarendon Press, 1959), pp. 130-132.
7. R. Byron Bird, Warren E. Stewart, and Edwin N. Lightfoot, *Transport Phenomena* (New York: John Wiley & Sons, Inc., 1960).
8. Frank Kreith, *Principles of Heat Transfer* (Scranton, Pennsylvania: International Textbook Company, 1965).
9. Thomas F. Fuller, Marc Doyle, and John Newman, "Simulation and Optimization of the Dual Lithium Ion Insertion Cell," *J. Electrochem. Soc.*, 141 (1994), 1-10.
10. William Tiedemann and John Newman, "Simulation of Electric Vehicle Driving Profiles Based on Mathematical Models of the Lead-Acid Battery System," Detroit meeting of the American Institute of Chemical Engineers, August 16-19, 1981. William Tiedemann and John Newman, "Mathematical Modeling of Phenomena Contributing to Thermal Rise in Lead-Acid Batteries Used in Electric Vehicles," *Proceedings of the Symposium on Advances in Lead-Acid Batteries*, pp. 360-377. Pennington, N. J.: The Electrochemical Society, 1984.

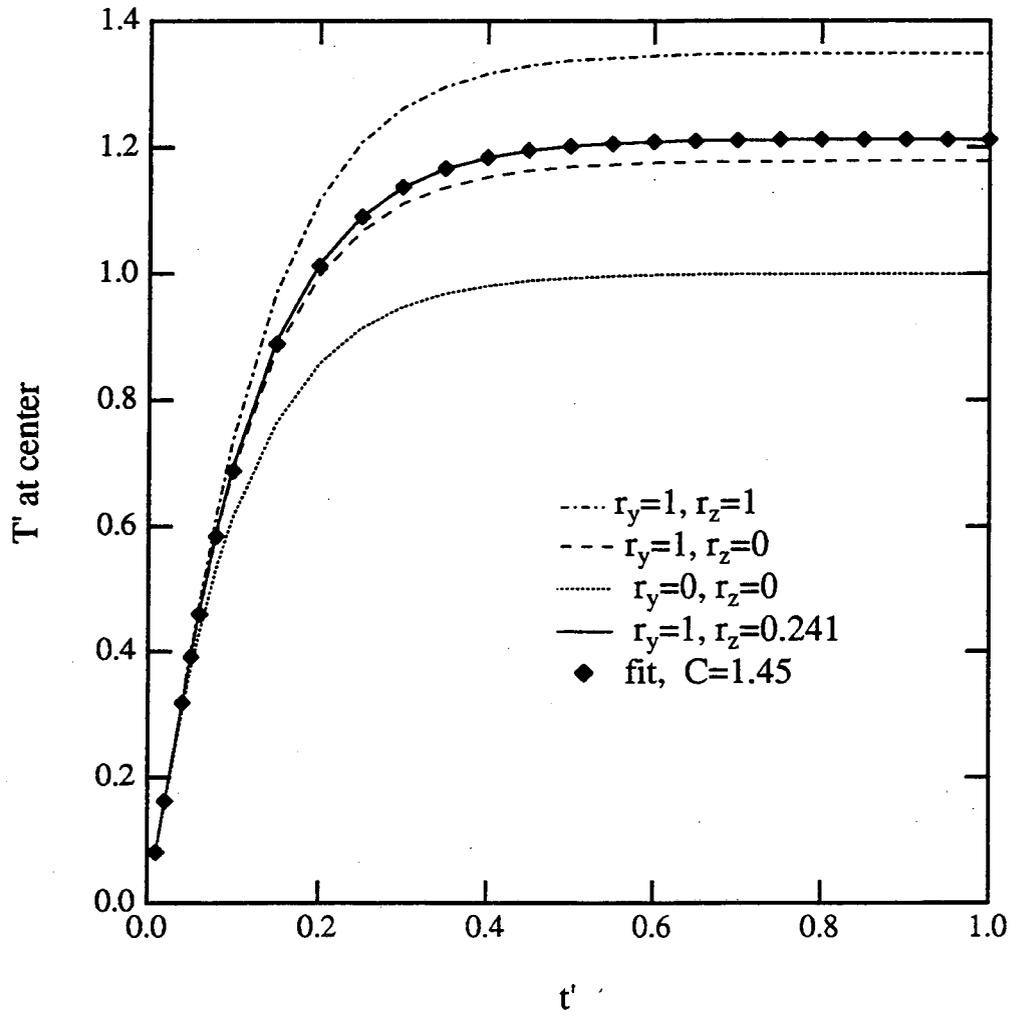


Figure 1. Transient temperature rise at the center of the module.

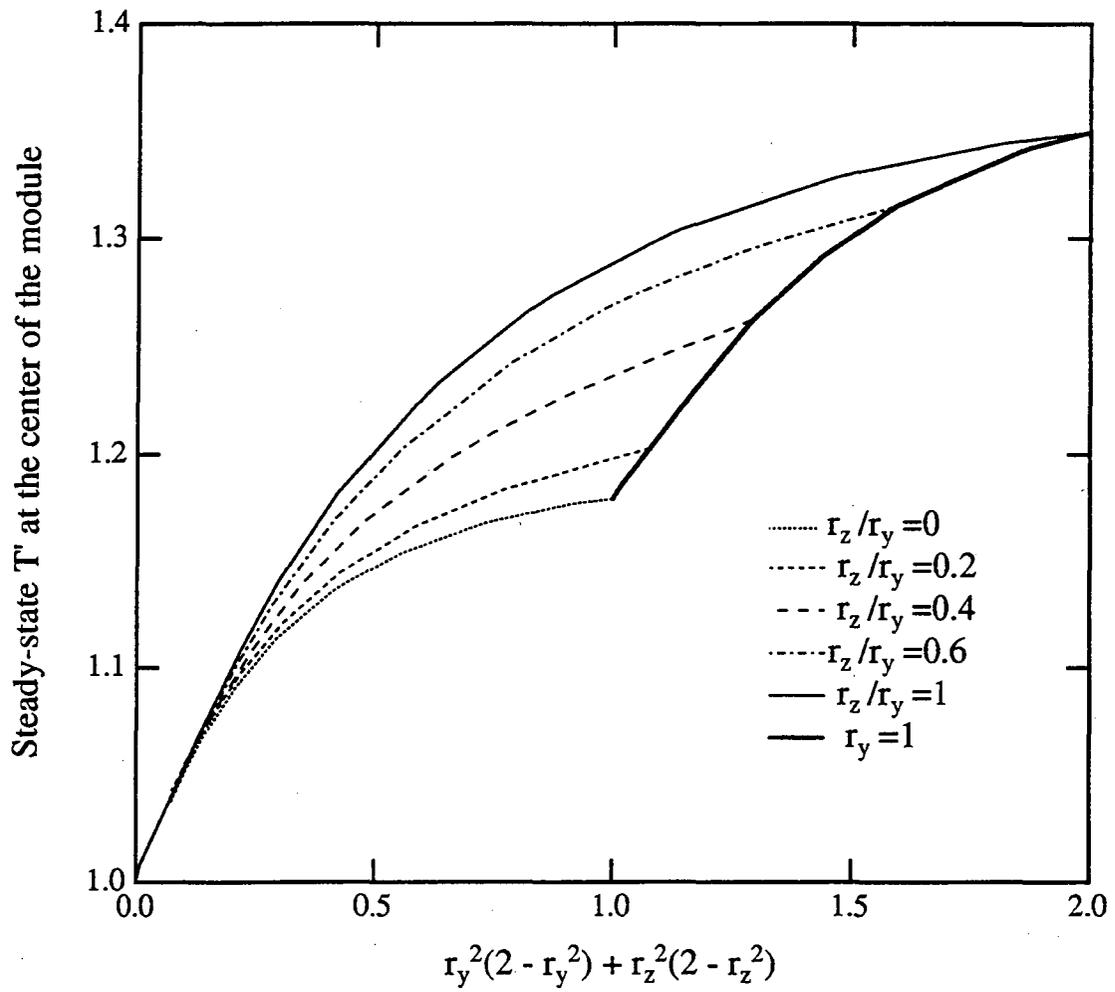


Figure 2. Dependence of steady-state temperature rise with the thermal aspect ratios.

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UNIVERSITY OF CALIFORNIA  
TECHNICAL INFORMATION DEPARTMENT  
BERKELEY, CALIFORNIA 94720

