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ESTIMATION OF THE CRITICAL RESOLVED SHEAR STRESS FOR DISLOCATION GLIDE THROUGH A RANDOM MIXTURE OF DISTINCT OBSTACLES

by

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ABSTRACT

Using techniques drawn from the statistical theory of branching processes, we approximate the critical resolved shear stress for the athermal planar glide of a dislocation, idealized as a flexible line of constant tension, through a random mixture of immobile point obstacles of distinct types. The approach simultaneously permits an estimate of the geometric properties of the particular obstacle configuration which determines the critical resolved shear stress. The estimates are shown to be in good agreement with empirical results obtained through direct computer simulation of glide through a random mixture of "strong" and "weak" points. A simple extension permits an estimate of the velocity of thermally activated glide through an array of mixed obstacle types at low temperature.

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I. INTRODUCTION

In a recent paper[1] we utilized techniques from the statistical theory of branching processes[2] to approximate the critical resolved shear stress for the athermal glide of a dislocation, idealized as a flexible line of constant tension, through a random array of identical, immobile obstacles which act as point barriers to glide. The approach simultaneously yielded a good approximation to the geometric properties of the particular dislocation configuration which determines the critical resolved shear stress for given obstacle strength. In the present paper we show how the approach may be generalized to treat glide through an array of obstacles having a distribution of properties.

The assumptions and basic equations used below are generalizations of those introduced in Reference 3 and reviewed in Reference 1. The glide plane of the dislocation is taken to be a square containing a random (Poisson) distribution of point obstacles whose density is given by the mean area (a) per point or by the characteristic length $\ell_s = a^{1/2}$. The area of the array may be written in dimensionless form: $A^* = A/a = n$ where n is the expected number of obstacles contained. To allow a distribution of obstacle properties, the obstacles are assumed to be randomly selected from a population containing p distinct types with x^α ($\alpha = 1, \ldots, p$) the fraction of obstacles of type α.

The dislocation is modeled as a flexible, extensible string of constant line tension, Γ, with a Burgers' vector of magnitude b in the glide plane. The resolved shear stress (τ) impelling glide is conveniently written in dimensionless form:

$$\tau^* = \tau \ell_s b/2\Gamma \tag{1.1}$$

If the dislocation encounters a configuration (i) of obstacles under stress τ^* it will take the form of a circular arc of dimensionless radius $R^*(= 1/(2\tau^*))$ between adjacent obstacles. If the distance between any two adjacent obstacles along (i) exceeds $2R^*$ or if the dislocation line anywhere intersects itself then configuration (i) is transparent to the dislocation and will be mechanically bypassed. If (i) is not transparent its mechanical stability is governed by a balance between the geometry of the configuration and the distribution of obstacle types along it.

At the kth obstacle on (i) the dislocation line forms the asymptotic angle $\psi_i^k (0 \leq \psi_i^k \leq \pi)$. The force, F_i^k, that the dislocation exerts on the kth obstacle on i is, in dimensionless form,

$$\beta_i^k = F_i^k/2\Gamma = \cos(\tfrac{1}{2}\psi_i^k) \tag{1.2}$$

whence $0 \leq \beta_i^k \leq 1$. Let the obstacle (k,i) be of type α and let the mechanical strength of an obstacle of type α be β_α, corresponding to the maximum force an obstacle of type α can sustain without being cut or locally bypassed. Then the dislocation is locally stable at (k,i) if

$$\beta^{*k}_i = \beta_i^k/\beta_\alpha < 1. \tag{1.3}$$

Evaluating the quantity β^{*k}_i for each obstacle on (i), with α given the value appropriate to the obstacle at (k,i), the condition for the mechanical stability of i becomes $\beta^{*k}_i < 1$ for all k on i, hence

$$\beta_i^* < 1 \tag{1.4}$$

where β_i^* is the maximum of the β^{*k}_i.

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The non-transparent configurations in a given array of obstacles are uniquely fixed by the applied stress, τ^*, as are the forces, β_i^k, exerted on them.[3] If τ^* is such that there is at least one configuration in the array over which the forces β_i^k satisfy the stability condition (1.4) then the dislocation will be mechanically pinned and can glide only with the help of thermal activation. The critical resolved shear stress for athermal glide, τ_c^*, is hence the minimum value of τ^* for which all configurations are mechanically unstable.

II. THE LIMITING CONFIGURATION IN AN ARRAY OF LIKE OBSTACLES

In Reference 1 we approximated the critical resolved shear stress τ_c^* for an arbitrarily large array of obstacles of like kind ($\beta_\alpha = \beta_c$) by identifying a "limiting configuration" which was necessarily at least as strong as the most stable configuration which might be encountered during glide. The derivation may be summarized as follows.

1) A stable dislocation configuration can be viewed as a chain of stable segments connecting the left hand side of the array to the right hand side. We may hence attempt to construct a stable configuration by beginning from the left hand side with a single segment and searching for segments which continue the chain across the array. If the applied stress is τ^* and the obstacle strength is β_c then a segment may be continued by connecting its right hand terminal point to any point found in the area shown in Figure 1, an area generated by rotating a circle of radius R^* through an angle $\theta_c (= 2 \sin^{-1}\beta_c)$ about the terminal point. The search area is parameterized by the coordinates (θ, ϕ) shown in

Figure 1. The lines of constant θ are generated by the leading edge of the circle as it is rotated; the lines of constant ϕ are generated by the trailing edge. The range of θ is chosen to be $0 \leq \theta \leq \theta_c$. The range of ϕ is taken to be $-\pi \leq \phi \leq \theta_c$. Points of the array are distributed over this search area with density 1, hence the number of points in the differential (dimensionless) area dA^* is simply dA^*, where

$$\begin{aligned} dA^* &= (R^*)^2 \sin(\theta - \phi) d\theta d\phi \\ &= (R^*)^2 \, da(\theta,\phi). \end{aligned} \tag{2.1}$$

Each point within the search area defines a possible segment continuing the chain.

2) The procedure for constructing a stable chain across the array by stochastically searching successive areas like that shown in Figure 1 bears a strong formal resemblance to the classical branching process in probability theory. The theory of branching processes[2] estimates the size of the kth generation of descent from a given initial event, assuming that the probability distribution for the number of descendants per parent event is known. A principal result of the theory is the extinction theorem, which states that if the expected number of descendants ($\langle n \rangle$) is less than one the line of descent will necessarily terminate after a finite number of generations. In the present problem it follows that τ^* and θ_c must be such that the search area A^* of Figure 1 is greater than one if it is to be possible to construct a stable chain across an array of arbitrarily large size.

3) The stable configurations of the dislocation are, however, additionally constrained in that the dislocation line cannot intersect

itself. A necessary (though not sufficient) condition[1] is that the expected value of the coordinate ϕ of the points used to extend the chain must be zero. Since the average value of ϕ over the search area (A^*, Figure 1) is less than zero (at least for $\theta_c < \pi$) the condition $\langle\phi\rangle = 0$ constrains the manner in which points may be selected from among those contained in A^*; points may only be used in subsets which have zero $\langle\phi\rangle$.

4) Given the considerations listed above, the points used to define the segments which extend a chain across the array are chosen according to a distribution function $f(\theta,\phi)$, $0 \leq f \leq 1$, which gives the fraction of the points found in $da(\theta,\phi)$ in an arbitrarily large sequence of searches of areas like A^* which are used to form the segments of a stable chain. The function $f(\theta,\phi)$ must satisfy at least the two constraints:

$$\langle n\rangle = (R^*)^2 \int_a f(\theta,\phi)da(\theta,\phi) > 1 \tag{2.2}$$

and

$$\langle\phi\rangle = (R^*)^2 \int_a \phi f(\theta,\phi)da(\theta,\phi) = 0 \quad . \tag{2.3}$$

For given values of $\langle n\rangle$ and the obstacle strength θ_c it may be shown (appendix to Ref. 1) that the radius R^* is minimized (τ^* maximized) under the constraint (2.3) if we choose

$$f(\theta,\phi) = \begin{cases} 1 & -\phi_0 \leq \phi \leq \theta_c \\ 0 & \phi \leq \phi_0 \end{cases} \tag{2.4}$$

where ϕ_0 is the solution of the equation

$$\langle\phi\rangle = \int_{a_0} \phi da(\theta,\phi) = \int_{-\phi_0}^{\theta_c} \phi da(\phi) = 0 \tag{2.5}$$

with the integral taken over the area a_0 shown in Figure 2.

If we now set

$$\begin{aligned} \langle n\rangle &= 1 \\ \langle\phi\rangle &= 0 \end{aligned} \tag{2.6}$$

and choose $f(\theta,\phi)$ according to equation (2.4) we obtain a value of τ^*, τ_0^*, which is necessarily larger than the stress necessary to pass the most stable chain which might be constructed across an arbitrarily large array of obstacles of strength θ_c. Hence

$$\tau_0^*(\theta) = \frac{1}{2}\left[\int_{a_0} da(\theta,\phi)\right]^{1/2} > \tau_c^*(\theta_c) \tag{2.7}$$

places an upper limit on the critical resolved shear stress.

The value τ_0^* is an upper bound on τ_c^* in two senses: first, by derivation, it is greater than any τ^* consistent with the inequality (2.2) and the constraint (2.3); second, it is the solution of an under-constrained problem in the sense that the condition $\langle\phi\rangle = 0$ is not sufficient to insure that the chain does not intersect itself (for example, $\langle\phi\rangle = 0$ over a configuration which approximates a figure eight) and in the further sense[1] that we have made an assumption of stochasticity which is not strictly true. It is, hence, not certain that τ_0^* yields a good estimate for τ_c^*. However, specific computer

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simulation of dislocation glide through arrays of moderate size (10^4 points[1] and up to 10^6 points[4]) has shown that τ_0^* is close to τ_c^* for obstacle strengths in the range $10^{-3} < \beta_c < 0.7$, a range which includes the obstacle strengths of physically realistic particles[5].

Given that τ_0^* approximates τ_c^*, the geometrical properties of the particular obstacle configuration which determines τ_c^* should be reasonably approximated by the geometrical properties of the "limiting configuration" which fixes τ_0^*. The latter can be computed from the expected distribution of points contained in the limiting search area of Figure 2.[1] A specific comparison between the theory and computer simulation results showed that the distribution of angles (forces) in the limiting configuration is almost precisely the same as the empirical distribution for the most stable configuration encountered in glide through an array of moderate size. Similarly, the mean segment lengths of the two configurations match closely. The limiting configuration is less successful in reproducing the distribution of segment lengths along the configurations obtained through computer simulation, though the finite size of the arrays employed in simulation makes this result difficult to interpret.

III. THE LIMITING CONFIGURATION IN AN ARRAY OF OBSTACLES OF DIFFERENT STRENGTHS

When the obstacles are not identical the procedure for generating the limiting configuration must be modified slightly. Let a stable chain be constructed left to right across an array which contains randomly distributed obstacles of p distinct types, labelled $\alpha = 1,\ldots,p$,

having fractions x^α and strengths β_α (or θ_α) as described in Section I. Consider the kth segment of the chain, which, in the language of the branching process, is a member of the kth generation of descent from the initial (or zeroth) segment. Let y_k^α be the probability that the kth segment terminates at an obstacle of type α; y^α is independent of k if k is large.

If the kth segment terminates at an α point then the obstacle defining the $(k + 1)^{th}$ segment must be chosen from among those located in the search area of an α-obstacle, an area like that shown in Figure 1 with $\theta_c = \theta_\alpha$. This area contains an expected number of points

$$n^\alpha = (A^*)^\alpha = (R^*)^2 a^\alpha \tag{3.1}$$

of which an expected fraction x^β are of type β. Let $f^{\alpha\beta}(\theta,\phi)$ be the fraction of the obstacles of type β found in the differential area $da(\theta,\phi)$ which are used to extend the chain from obstacles of type α. Then the expected number of descendents (stable segments) in the $(k + 1)^{th}$ generation per point in the kth generation is

$$\langle n\rangle = (R^*)^2 \sum_\alpha y_k^\alpha \int_{a^\alpha} \left[\sum_\beta f^{\alpha\beta}(\theta,\phi)x^\beta\right] da(\theta,\phi) \ . \tag{3.2}$$

By the extinction theorem of branching processes $\langle n\rangle$ must be greater than one if the chain is to extend across an array of arbitrarily large size. The expected value of the coordinate ϕ is

$$\langle\phi\rangle = (R^*)^2 \sum_\alpha y_k^\alpha \int_{a^\alpha} \left[\sum_\beta f^{\alpha\beta}(\theta,\phi)x^\beta\right] \phi\, da(\theta,\phi) \ , \tag{3.3}$$

which must vanish if the chain is not to intersect itself. The expected

fraction of segments of the $(k + 1)^{th}$ generation which terminate at obstacles of type α is

$$y^{\alpha}_{k+1} = \langle n\rangle^{-1}(R^*)^2 \sum_{\beta} y^{\beta}_k \int_{a^{\beta}} f^{\beta\alpha}(\theta,\phi)x^{\alpha}da(\theta,\phi) \ . \qquad (3.4)$$

When k is large,

$$y^{\alpha}_{k+1} = y^{\alpha}_k = y^{\alpha} \qquad (3.5)$$

The limiting configuration is obtained from equations (3.2)-(3.5) by setting $\langle n\rangle = 1$ and $\langle\phi\rangle = 0$, and then choosing the fractions y^{α} and functions $f^{\alpha\beta}(\theta,\phi)$ so that R^* is minimized (τ^* maximized) for a given set of fractions x^{α} and strengths θ_{α}. With $\langle n\rangle = 1$ and $\langle\phi\rangle = 0$, equations (3.2)-(3.4) may be conveniently rewritten

$$(2\tau^*) = \int_{a^s} f(\theta,\phi)da(\theta,\phi) \qquad (3.6)$$

$$0 = \int_{a^s} \phi f(\theta,\phi)da(\theta,\phi) \qquad (3.7)$$

$$y^{\alpha} = x^{\alpha}\left\{\frac{1}{(2\tau^*)^2}\sum_{\beta} y^{\beta}\int_{a^{\alpha}} f^{\beta\alpha}(\theta,\phi)da(\theta,\phi)\right\} \qquad (3.8)$$

where a^s is the search area of the obstacle of greatest strength (maximum $\theta_{\alpha} = \theta_s$) and where

$$f(\theta,\phi) = \sum_{\alpha,\beta} f^{\alpha\beta}(\theta,\phi)x^{\beta}y^{\alpha}h^{\alpha}(\theta) \le \sum_{\alpha} y^{\alpha}h^{\alpha}(\theta) \qquad (3.9)$$

with

$$h^{\alpha}(\theta) = \begin{cases} 1 & 0 \le \theta \le \theta_{\alpha} \\ 0 & \theta > \theta_{\alpha} \end{cases} \ . \qquad (3.10)$$

It follows directly from the theorem given in the Appendix to Reference 1 that the integral on the right hand side of equation (3.6) is maximized under the constraint (3.7) if $f(\theta,\phi)$ is assigned the value

$$f(\theta,\phi) = \begin{cases} \sum_\alpha y^\alpha h^\alpha(\theta) & -\phi_0 \leq \phi \leq \theta_s \\ 0 & \text{otherwise} \end{cases} \tag{3.11}$$

where ϕ_0 is the solution to the equation

$$0 = \left\{ \int_{-\phi_0}^{\theta_c} \int_0^{\theta_c} \phi \left\{ \sum_\alpha y^\alpha h^\alpha(\theta) \right\} da(\theta,\phi) \right\} . \tag{3.12}$$

Equation (3.8) then yields the identity

$$y^\alpha = x^\alpha \tag{3.13}$$

and equations (3.6) and (3.7) may be written in the more compact form

$$(\tau_0^*)^2 = \frac{1}{4} \sum_\alpha x^\alpha a_0^\alpha \tag{3.14}$$

$$0 = \sum_\alpha x^\alpha a_0^\alpha \langle\phi\rangle^\alpha \tag{3.15}$$

where τ_0^* is the strength of the limiting configuration (an upper limit on τ_c^*), a_0^α is the sub-area of a^α over which $-\phi_0 \leq \phi \leq \theta_\alpha$, and $\langle\phi\rangle^\alpha$ is the average value of ϕ over a_0^α.

The sequential solution of equations (3.15) and (3.14) determines τ_0^*, which estimates the critical resolved shear stress, τ_c^*, for glide through an arbitrarily large array of randomly distributed obstacles having strengths θ_α ($\alpha = 1,\ldots,p$) and fractions x^α. The properties of

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the particular configuration which determines τ_c^* may also be approximated by the properties of the limiting configuration. Three properties are of particular interest: the fraction, c^α, of obstacles of type α in the configuration, the distribution of angles (θ) or, equivalently, of forces (β) along it, and the mean value of the separation between adjacent obstacles.

The fraction c^α is easily computed. The kth generation of descent from an initial segment contains an expected fraction x^α of segments which terminate at obstacles of type α. An expected fraction $R^{*2}a_0^\alpha$ of these are continued by stable setments to points found within the optimal search area, and hence become "parents" of the $(k + 1)^{th}$ generation. Since successive generations of the limiting configuration are stochastically independent, obstacles of type α will appear in the limiting configuration in precisely the fraction in which they are expected as "parents" of the $(k + 1)^{th}$ generation. Hence

$$c^\alpha = x^\alpha a_0^\alpha R^{*2} \tag{3.16}$$

The computation of the distribution of forces in the limiting configuration is also straight-forward, given the discussion in Reference 1. The fraction of obstacles of type α along the chain is given by equation (3.16). It follows from equations (3.7)-(3.9) of Reference 1 that, if normalization is properly accounted for, the distribution of forces on obstacles of type α is specified by the density function

$$\rho^\alpha(\beta) = \begin{cases} (a_0^\alpha R^{*2})^{-1}\rho(\beta;\tau^*) & 0 \le \beta \le \beta_\alpha \\ 0 & \beta > \beta_\alpha \end{cases} \tag{3.17}$$

where $\rho(\beta;\tau^*)$ is the density function given by equation (3.9) of Reference 1:

$$\rho(\beta;\tau^*) = 2R^{*2}\{(1-\beta^2)^{-1/2}[1-(1-2\beta^2)\cos\phi_0] + 2\beta \sin \phi_0\} \tag{3.18}$$

The density of forces in the limiting configuration is hence

$$\rho(\beta) = \sum_\alpha c^\alpha \rho^\alpha(\beta) = \rho(\beta;\tau^*) \sum_\alpha x^\alpha h^\alpha(\beta) \tag{3.19}$$

where $h^\alpha(\beta)$ is a weighting function equal to one if $\beta \leq \beta_\alpha$ and to zero otherwise.

A similar argument yields the equation for the mean spacing between adjacent obstacles along the limiting configuration:

$$<\ell> = \sum_\alpha x^\alpha <\ell(\tau^*)>^\alpha \quad , \tag{3.20}$$

where the function $<\ell(\tau^*)>^\alpha$ is the form appropriate to (α) of the function given by equation (3.14) of Reference 1:

$$<\ell(\tau^*)>^\alpha = (\tfrac{2}{3})(2R^*)^3\left\{\cos(\theta_\alpha/2)[1 - \tfrac{1}{3}\cos^2(\theta_\alpha/2)] - \cos\left(\frac{\theta_\alpha+\phi_0}{2}\right)\left[1 - \frac{1}{3}\cos\left(\frac{\theta_\alpha+\phi_0}{2}\right)\right]\right\} \tag{3.21}$$

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IV. COMPARISON WITH COMPUTER SIMULATION RESULTS

To test the accuracy of the equations developed in the previous section we performed computer simulation experiments on glide through arrays containing obstacles of two distinct kinds. Given the characteristics of the simulation code employed,[6] which was written to simulate thermally activated glide at constant stress, we used an indirect procedure to obtain numerical data for comparison with the theory. Specifically, we chose the strengths β_s and β_w, and fractions, x^s and $x^w (= 1-x^s)$, of the strong and weak obstacles, and then computed the limiting stress τ_0^* from equations (3.14) and (3.15). We then set the stress $\tau^* = \tau_0^*$ and simulated glide through an array of 1200 obstacles, a random fraction x^s of which were taken to be strong. If the dislocation is allowed to move through the array along a path found by passing each non-transparent configuration i at the obstacle at which β_i^{*k} (equation (1.3)) has its maximal value (β_i^*) then the dislocation will necessarily encounter the most stable configuration within the array (this path is the analogue of the "minimal angle" path[3] through an array of like obstacles). The theory developed here approximates the strength of this configuration, in an array of large size, as $\beta_1^* = 1.0$. The computer simulation yields an empirical value of β_1^* for a particular array.

In the present program we tested three combinations of obstacle strengths at each of three choices of the fraction x^s, giving the total of nine cases listed in Table I. For each case we simulated glide through ten arrays of 1200 points. Given the finite size of these arrays, there is some scatter in the β_1^* values found and a statistical bias toward $\beta_1^* < 1.0$ which becomes more pronounced as the array size or the

applied stress is decreased. Nontheless the theory appears to give a good estimate of the critical resolved shear stress τ_c^* (as measured by the agreement $\beta_1^* \sim 1.0$), the fraction c^w of weak obstacles in the configuration which determines τ_c^*, and the mean value of the interobstacle spacing along this configuration.

To test the accuracy of the force distribution predicted by equation (3.17) we determined an empirical force distribution for case 7 by compiling the forces (β) along the most stable configurations in each of twenty-five arrays of 1200 points. The resulting normalized histogram is compared to the theoretical prediction in Figure 3. The fit seems good. It should be noted that the fit requires simultaneous estimates of the strength of the most stable configuration, the fraction of weak obstacles along it, and the distribution of forces given those parameters.

V. EXTENSION TO THERMALLY ACTIVATED GLIDE

The approach developed in Section III may also be used to estimate the velocity of thermally activated glide at low temperature. As was discussed in Reference 3, when the temperature is sufficiently low the expected time required for the dislocation to transit the array, and hence the velocity of glide, is essentially determined by the expected time for the dislocation to activate past the most stable configuration it encounters. This activation time is itself determined by the expected time for activation at the point along the most stable configuration which offers the minimum activation barrier. Defining the dimensionless time

$$t^* = \nu t \tag{5.1}$$

where ν is the frequency with which the dislocation attempts thermal activation at an obstacle (assumed constant) and the dimensionless velocity

$$\langle v^* \rangle = n^{1/2} \langle a^* \rangle \tag{5.2}$$

where n is the number of points in the array and $\langle a^* \rangle$ is the dimensionless area swept out per unit t^*, it follows[3] that as T approaches zero

$$\langle v^* \rangle \simeq n^{1/2} \exp(-\Delta G_1/kT) \tag{5.3}$$

where ΔG_1 is the minimum of the activation barriers associated with the obstacles in the most stable configuration. Computer simulation studies[7] have shown that equation (5.3) gives a reasonable approximation to the glide velocity over a wide range of temperature.

The activation barrier ΔG at an obstacle depends[8,3] on the dislocation configuration at the obstacle (hence on β, or, equivalently on θ) and on the nature of the dislocation-obstacle interaction. If the dislocation-obstacle interaction is reasonably simple then the activation energy may be written as a function ($\Delta G(\beta)$) of the force on the obstacle. Inverting this function gives

$$\beta_\alpha = \beta_\alpha(\Delta G) \tag{5.4}$$

or, equivalently,

$$\theta_\alpha = \theta_\alpha(\Delta G), \tag{5.5}$$

determining the force, β, or angle, θ, associated with a particular value of the activation energy for a particular type (α) of obstacle.

When conditions are such that equations (5.3) and (5.4) are obeyed the equations presented in Section III may be used to estimate the velocity of glide as a function of stress and temperature. We require the function $\Delta G_1(\tau^*)$, the minimum activation energy in the most stable configuration encountered in glide at stress τ^*.

Consider an array which contains a fraction x^α of obstacles of type α ($\alpha = 1,\ldots,n$). Assume that equation (5.5) is obeyed for each obstacle. Then the stress (τ^*) at which the most stable configuration encountered poses an activation barrier ΔG_1 is the maximum stress at which there exists a configuration satisfying

$$\theta_\alpha \leq \theta_\alpha(\Delta G_1) \tag{5.6}$$

for all α. The stress $\tau^*(\Delta G_1)$ may be approximated by employing the values $\theta_\alpha(\Delta G_1)$ in place of θ_α in equations (3.14) and (3.15). Inverting this function gives $\Delta G_1(\tau^*)$ and consequently $\langle v^*(\tau^*,T)\rangle$ over the range of conditions for which equation (5.3) holds.

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Table I. A comparison of the predicted values of β_1^* (= 1.0), c^w, and $\langle \ell \rangle$ with the mean of the values found through computer simulation for the most stable configuration in each of ten arrays of 1200 points under each of the nine conditions shown.

Case	β_s	β_w	x^w	τ_0^*	$\langle \beta^* \rangle_{exp}$	c^w_{theory}	$\langle c^w \rangle_{exp}$	ℓ_{theory}	$\langle \ell \rangle_{exp}$
1	0.5	0.2	0.5	0.240	1.006	0.13	0.13	1.30	1.28
2	0.5	0.05	0.5	0.231	0.998	0.020	0.021	1.44	1.46
3	0.2	0.05	0.5	0.057	0.919	0.064	0.071	2.25	2.08
4	0.5	0.2	0.17	0.299	0.997	0.030	0.041	1.11	1.05
5	0.5	0.05	0.17	0.297	1.002	0.0042	0.0048	1.13	1.12
6	0.2	0.05	0.17	0.073	0.948	0.014	0.015	1.84	1.73
7	0.5	0.2	0.83	0.157	0.969	0.40	0.44	1.65	1.59
8	0.5	0.05	0.83	0.135	1.007	0.088	0.145	2.31	2.26
9	0.2	0.05	0.83	0.035	0.890	0.24	0.23	3.21	3.01

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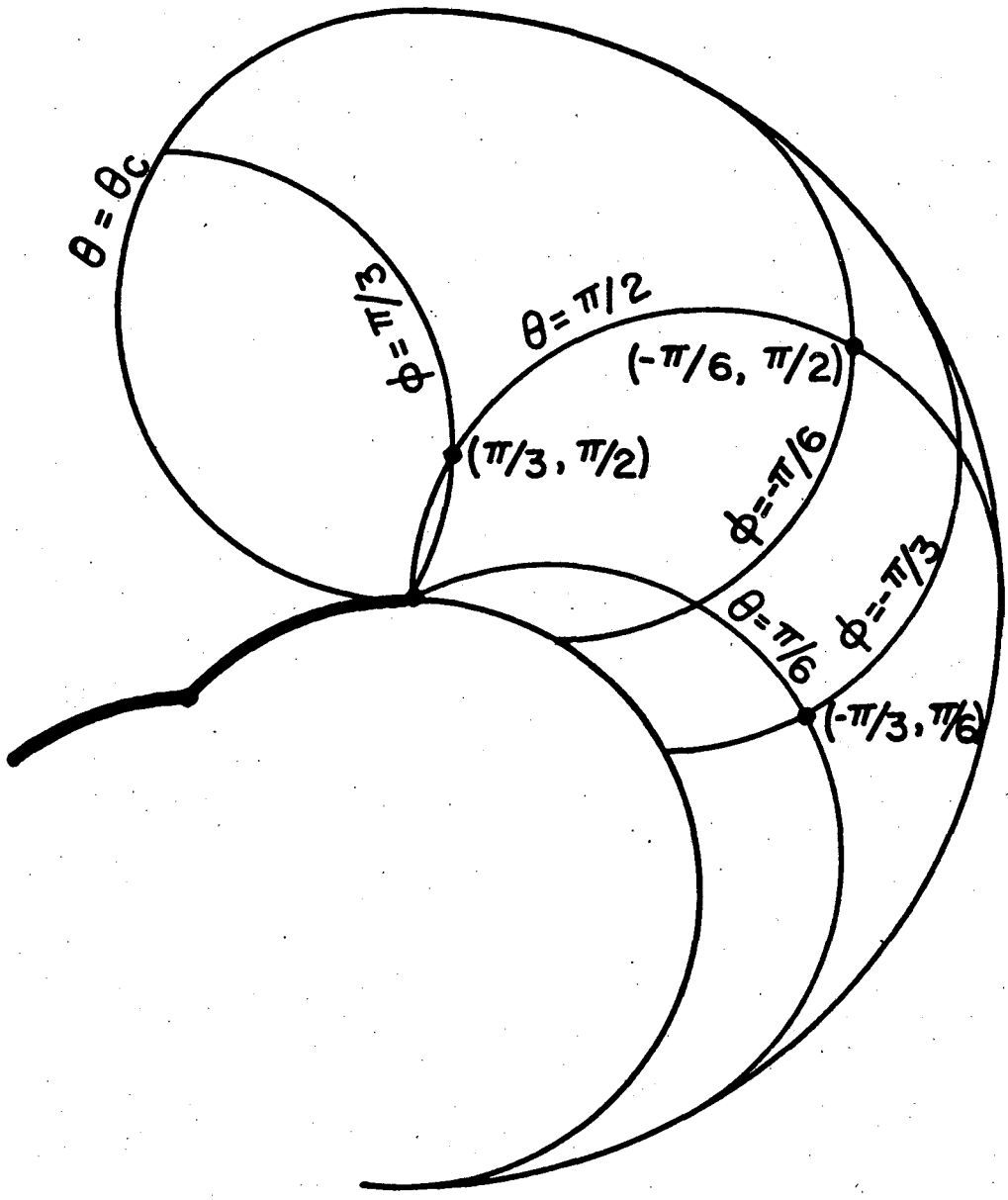

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Figure 1. The valid search area for extending a chain of stable dislocation segments given in obstacle strength θ_c.

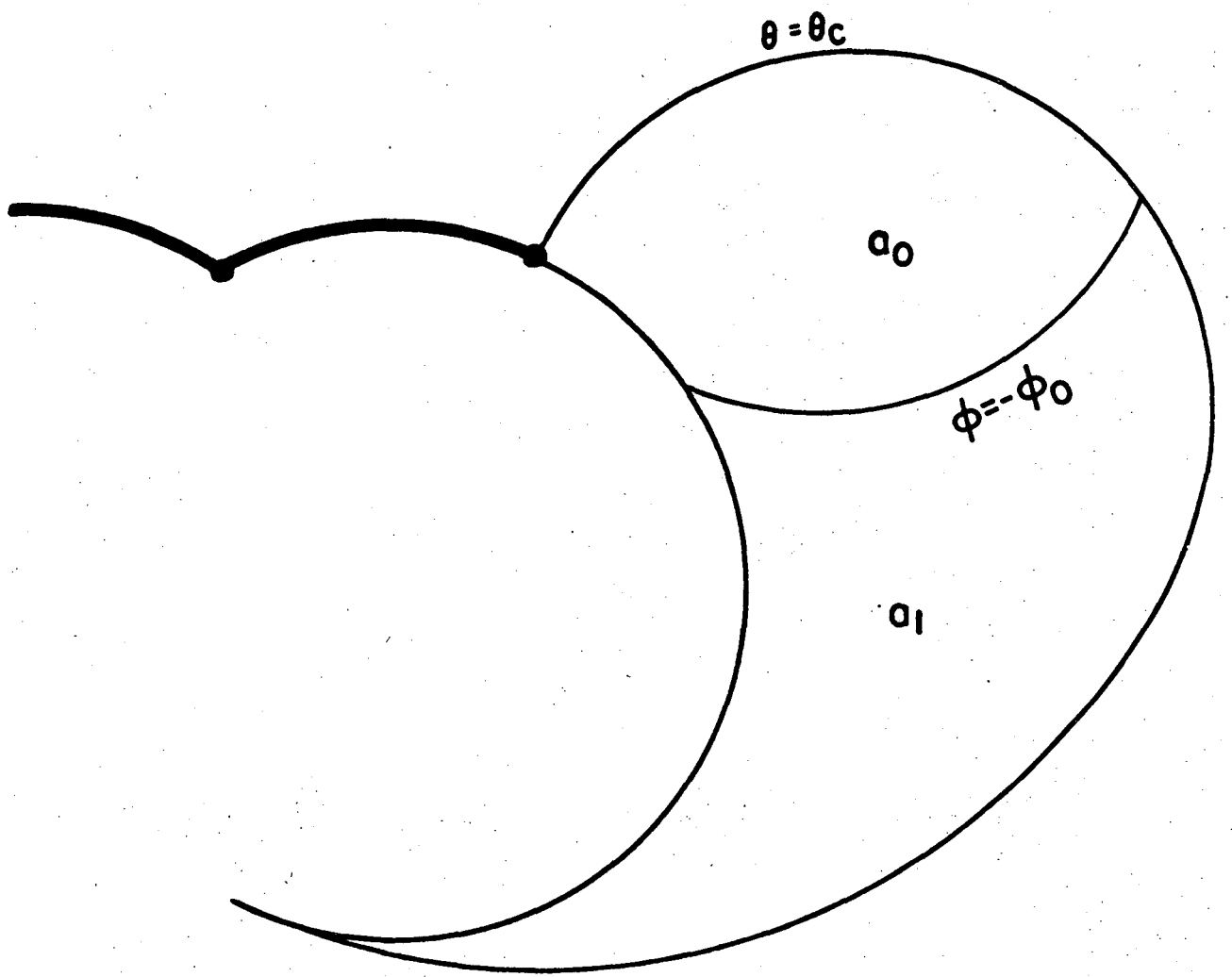

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Figure 2. The optimal search area, a_0, for extending a chain of stable dislocation segments given an obstacle strength θ_c.

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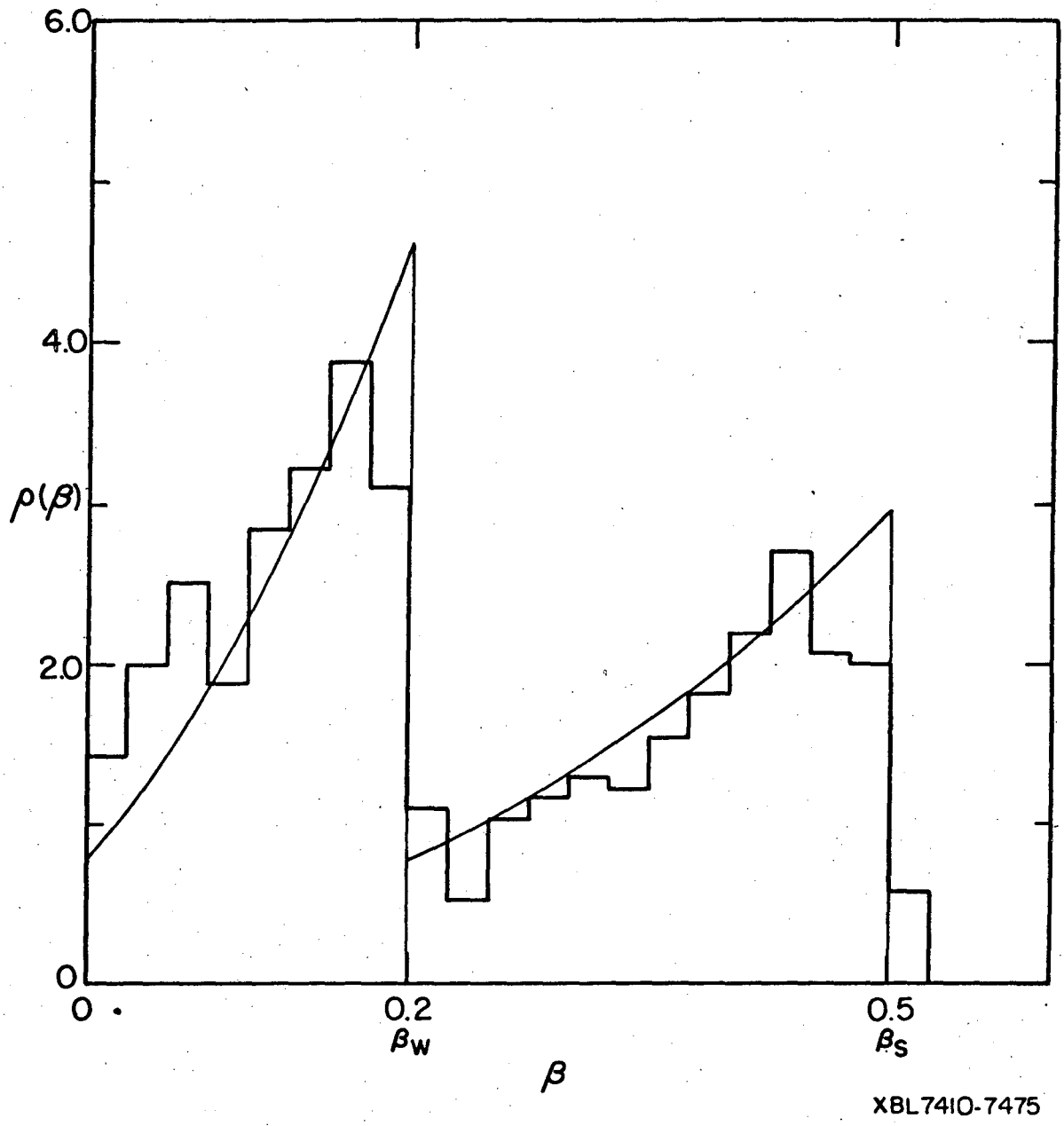

Figure 3. The theoretical distribution of forces (β) in the limiting configuration for case number 7 (Table I) compared to an empirical histogram obtained through computer simulation as described in the text.

0 0 0 0 4 2 0 7 4 4 0

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