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HIGH-ENERGY HADRON-DEUTERON SCATTERING

C. J. Joachain and C. Quigg

June 15, 1970

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HIGH-ENERGY HADRON-DEUTERON SCATTERING

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HIGH-ENERGY HADRON-DEUTERON SCATTERING

C. J. Joachain*

Department of Physics
University of California
Berkeley, California 94720

and

C. Quigg†

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

June 15, 1970

ABSTRACT

We review theoretical descriptions of hadron-deuteron scattering at high energies. All the specific models have in common the Glauber approximation as their principal ingredient. We devote considerable effort to a discussion of forward elastic scattering, for which the theories have reached the highest degree of refinement. We detail the modifications to the simple theory which have emerged from fine structure in

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† Supported by the United States Atomic Energy Commission. Address after 15 September 1970: Institute for Theoretical Physics, State University of New York, Stony Brook, Long Island, New York 11790.

experimental cross sections. The relation between the Glauber picture and Regge theory is analyzed, and the present incompleteness of both approaches is exposed. Brief remarks are made on the anomalous thresholds in energy variables which occur for loosely bound systems, such as the deuteron. Finally we discuss more basic generalizations of the Glauber method to include inelastic scattering of the projectile or of one of the constituent nucleons within the deuteron. A few specific experimental tests are listed. By formulating diffractive excitation of the projectile in terms of the multiple scattering series we show how unstable hadron-nucleon elastic scattering may be explored.

Introduction

A long-standing problem of great importance, hadron deuteron scattering at high energies has recently attracted considerable interest, both theoretical and experimental. Its sources of appeal are manifold and range from the practical--determinations of neutron cross sections--to the esoteric--investigations of the properties of Regge cuts. The potential, and by now partially realized, utility of deuteron targets has been a stimulus for the development of techniques for extracting hadron-nucleon scattering amplitudes from hadron-deuteron data. Such techniques have reached a high level of sophistication for forward elastic scattering, for which experiments have been the most numerous and detailed. Lying on the borderline between elementary particle physics and nuclear physics, hadron-deuteron scattering has been a locus of fruitful interaction between the two fields.

In this review we consider hadron-deuteron scattering from two points of view. The first of these, namely the high-energy diffraction theory pioneered by Glauber, is discussed in the first part of the article. We begin in Sec. 1.1 with a summary of the eikonal approximation, first for two-body collisions and then, by analogy with the two-body case, for hadron-nucleus scattering. The approximations which enter into the derivation of Glauber theory are discussed in detail. Section 1.2 is devoted to the application of Glauber's high-energy diffraction theory to hadron-deuteron collisions. Particular emphasis is given to elastic scattering, for which a comprehensive comparison of recent theoretical and experimental work is made. Various generalizations of the Glauber formalism are discussed in Sec. 1.3.

In the second part we discuss hadron-deuteron collisions from the point of view of Regge theory. Section 2.1 deals with the angular momentum structure of Glauber theory. Beginning with a study of the connection between diffraction scattering and Regge poles, we go on to investigate the Regge cut contained in the Glauber eclipse term in the light of existing knowledge about Regge cuts. In Sec. 2.2 we deal very briefly with anomalous singularities in the dnp vertex function. Our discussion then turns to phenomenological applications, and in Sec. 2.3 we propose some experiments which will be of value in learning how to generalize the theoretical description to inelastic collisions.

1. Hadron-Deuteron Scattering in the Glauber Formalism

In this first part we discuss hadron-deuteron collisions in the framework of the high-energy diffraction theory. The basic contribution is due to Glauber (1953, 1955, 1959, 1960, 1967, 1969) who generalized the classical Fraunhofer diffraction theory (see, for example, Born and Wolf, 1964) and the eikonal approximation of Molière (1947) to deal with high-energy hadron-nucleus processes.

1.1. The Eikonal Approximation

(a) Two-body scattering

As an introduction to the techniques used in high-energy diffraction theory, let us first consider the case of two-body scattering. For high-energy, small angle collisions, it is particularly convenient to discuss the scattering in the eikonal approximation (Molière, 1947; Glauber, 1953, 1955, 1959; Schwinger, 1954; Malenka 1954; Schiff, 1956; Saxon and Schiff, 1957). Let $\underset{\sim}{r}$ be the relative coordinate of the two particles, whereas $\underset{\sim}{k}_i$ and $\underset{\sim}{k}_f$ are the initial and final relative momenta. If the z axis is chosen along the incident direction, the elastic scattering amplitude $f(\theta,\phi)$ may be expanded in spherical harmonics as

$$f(\theta,\phi) = \frac{1}{2ik_i} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} (2\ell + 1)(S_{\ell m} - \delta_{m0})\, Y_{\ell m}(\theta,\phi), \tag{1.1}$$

where the transmission coefficients $S_{\ell m}$ are the S-matrix elements in the angular momentum representation. We now decompose the vector $\underset{\sim}{r}$ as

$$\underset{\sim}{r} = \underset{\sim}{b} + z\,\hat{k}_i\,, \tag{1.2}$$

where $\underset{\sim}{b}$ is the impact parameter vector, of length $b \simeq \frac{1}{k}(\ell + \frac{1}{2})$, perpendicular to the incident direction. Since many partial waves contribute to the scattering at high energies, the summation over ℓ in Eq. (1.1) may be transformed into an integral over b. The additional small angle condition then allows one to neglect the longitudinal momentum transfer and to obtain the eikonal representation of the scattering amplitude as (Franco and Glauber, 1966)

$$f(\underset{\sim}{k}_i,\underset{\sim}{k}_f) = \frac{ik_i}{2\pi}\int d^2\underset{\sim}{b}\; e^{i(\underset{\sim}{k}_i-\underset{\sim}{k}_f)\cdot\underset{\sim}{b}}[1 - e^{i\chi(\underset{\sim}{b})}], \tag{1.3}$$

where $\chi(\underset{\sim}{b})$ is the phase-shift function. For potentials which enjoy azimuthal symmetry so that $S_{\ell m} = \delta_{m0}\exp(2i\,\delta_\ell)$, one readily obtains from Eq. (1.3) the Fourier-Bessel representation of the scattering amplitude

$$f(\underset{\sim}{k}_i,\underset{\sim}{k}_f) = ik_i\int_0^\infty db\cdot b\,J_0(qb)[1 - e^{i\chi(b)}], \tag{1.4}$$

where q is the magnitude of the momentum transfer vector $\underset{\sim}{q} = \underset{\sim}{k}_i - \underset{\sim}{k}_f$. Defining the quantity

$$\Gamma(\underset{\sim}{b}) = 1 - e^{i\chi(\underset{\sim}{b})}, \tag{1.5}$$

which will frequently be used below, we may rewrite Eq. (1.3) as

$$f(\underset{\sim}{k}_i,\underset{\sim}{k}_f) = \frac{ik_i}{2\pi}\int d^2\underset{\sim}{b}\; e^{i(\underset{\sim}{k}_i-\underset{\sim}{k}_f)\cdot\underset{\sim}{b}}\;\Gamma(\underset{\sim}{b})\,. \tag{1.6}$$

It is important to note that the derivation of Eq. (1.3) does not require the existence of a potential to describe the collision process, although an "optical" potential can always be found to describe the scattering in the eikonal approximation (Glauber, 1959; Omnès, 1965). Given such a potential $V(\underset{\sim}{r})$, one can easily derive the eikonal scattering amplitude (1.3) by using the fact that the classical trajectories are almost straight lines in the incident direction. Stationary-phase arguments (Schiff, 1956) or the fact that the incoming wave is modulated by a function which varies slowly over the de Broglie wavelength of the incident hadron (Glauber, 1959) then yield Eq. (1.3) with a phase shift function given by

$$\chi(\underset{\sim}{b}) = -\frac{1}{v_i}\int_{-\infty}^{+\infty} V(\underset{\sim}{b},z)\, dz, \tag{1.7}$$

where v_i is the relative initial velocity. For a detailed discussion see Glauber (1959). Another interesting way of deriving the eikonal result (1.3) is to examine the free propagator $G_0(\underset{\sim}{r},\underset{\sim}{r}')$ appearing in the Lippmann-Schwinger equation for scattering by a potential $V(\underset{\sim}{r})$. (Malenka, 1954; Schiff, 1956). Let us write this propagator in momentum space as $(\hbar = c = 1)$,

$$G_0(\underset{\sim}{r},\underset{\sim}{r}') = -(2\pi)^{-3}\, 2E \int \frac{e^{i\underset{\sim}{\kappa}\cdot(\underset{\sim}{r}-\underset{\sim}{r}')}}{\kappa^2 - k_i^2 - i\epsilon}\, d^3\underset{\sim}{\kappa}\,, \tag{1.8}$$

where E is the total (relativistic) energy in the center-of-mass system. Let us change the variable of integration in Eq. (1.8) to

$$\underset{\sim}{Q} = \underset{\sim}{\kappa} - \underset{\sim}{k}_i\,. \tag{1.9}$$

If we take into account the fact that the momentum transfers are small in a high-energy small-angle collision we may "linearize" the denominator of the integrand--i.e., neglect the Q^2 term--and write

$$G_0(\underset{\sim}{r},\underset{\sim}{r}') \simeq -(2\pi)^{-3}\, 2E\, e^{i\underset{\sim}{k}_i\cdot(\underset{\sim}{r}-\underset{\sim}{r}')} \int \frac{e^{i\underset{\sim}{Q}\cdot(\underset{\sim}{r}-\underset{\sim}{r}')}}{2\underset{\sim}{k}\cdot\underset{\sim}{Q} - i\epsilon}\, d^3\underset{\sim}{Q} \quad . \tag{1.10}$$

Using the decomposition (1.2) of $\underset{\sim}{r}$ and a similar one for $\underset{\sim}{r}'$, we then obtain the approximate formula

$$G(\underset{\sim}{r},\underset{\sim}{r}') \simeq \begin{cases} -i\, E\, k_i^{-1}\, e^{i\underset{\sim}{k}_i\cdot(\underset{\sim}{r}-\underset{\sim}{r}')}\, \delta^2(\underset{\sim}{b} - \underset{\sim}{b}'), & z > z' \\ 0 & , \; z < z', \end{cases} \tag{1.11}$$

which clearly exhibits forward propagation between successive interactions with the potential. This linearized propagator leads directly to the eikonal scattering amplitude (1.3) with a phase-shift function given by Eq. (1.7). Incidentally let us remark that the importance of the four-dimensional version of the linearized propagator in treating field theoretical problems was recognized by Schwinger (1954) and used recently by several authors (Chang and Ma, 1969; Abarbanel and Itzykson, 1969; Lévy and Sucher, 1970; Englert, Nicoletopoulos, Brout, and Truffin, 1969) to sum the series of Feynman amplitudes corresponding to large classes of ladder diagrams.

The basic Eq. (1.3) has been derived in the c.m. system of the two colliding particles. However, because of the simple kinematics of high-energy small-angle scattering, the expression (1.3) retains its validity in the laboratory system. The only modification is that $\underset{\sim}{k}_i$ and $\underset{\sim}{k}_f$ now represent the initial and final momenta of the projectile

in the laboratory frame. Of course, the magnitude of $\underset{\sim}{k}_f$ is now smaller than that of $\underset{\sim}{k}$ because of recoil effects, but these effects are small for scattering near the forward direction and can be minimized by interpreting the quantity $(-q^2)$ as the Mandelstam variable t, namely the square of the four-momentum transfer of the collision. In what follows, we shall mainly use the laboratory frame, since we want the target nuclei to remain nonrelativistic.

(b) Hadron-nucleus collisions

Consider now a hadron X of initial laboratory energy E_i and momentum $\underset{\sim}{k}_i$ incident on a nucleus of mass number A. Assuming that the high-energy small-angle conditions are satisfied, the eikonal method can be generalized in the following way (Glauber, 1953, 1955, 1959). Since the incident hadron travels much faster than the characteristic nuclear frequencies, it is reasonable to suppose that the target nucleons are "frozen" during the passage of the projectile through the nucleus. In addition, if one assumes that the incident particle interacts with the target nucleons via two-body spin-independent* interactions, the transition amplitude from an initial nuclear state $|i\rangle$ to a final nuclear state $|f\rangle$ is given by the overlap integral

$$F_{fi} = \frac{ik_i}{2\pi} \langle f| \int d^2\underset{\sim}{b}\, e^{i(\underset{\sim}{k}_i-\underset{\sim}{k}_f)\cdot\underset{\sim}{b}} [1-e^{i\chi_{tot}(\underset{\sim}{b};\underset{\sim}{r}_1\cdots\underset{\sim}{r}_A)}]|i\rangle, \quad (1.12)$$

---

* The generalization to spin-dependent interactions will be discussed below.

where

$$\underset{\sim}{r} = \underset{\sim}{b} + z\,\hat{k}_i \tag{1.13}$$

is the coordinate of the incoming hadron,

$$\underset{\sim}{r}_j = \underset{\sim}{b}_j + z_j\,\hat{k}_i \tag{1.14}$$

are the coordinates of the target nucleons, and

$$\chi_{tot}(\underset{\sim}{b};\underset{\sim}{r}_1,\cdots,\underset{\sim}{r}_A) = \sum_{j=1}^{A} \chi_j(\underset{\sim}{b} - \underset{\sim}{b}_j) \tag{1.15}$$

is the sum of the phase shifts contributed by each of the target nucleons.

The crucial property of phase-shift additivity, expressed by Eq. (1.15), is clearly a direct consequence of the one-dimensional nature of the relative motion, together with the neglect of three-body forces, target nucleon motion, and longitudinal momentum transfer.

Another important remark concerning Eq. (1.12) is that it applies only to collisions for which the energy transfer ΔE is very small compared with the incident energy E_i. This is true for elastic collisions and "mildly" inelastic ones in which the nucleus is excited or perhaps breaks up. It is <u>not</u> true for "deeply" inelastic collisions, in which the nature of the incident or target hadrons is modified or the number of particles undergoing the collision is altered. Leaving aside such inelastic processes, we return to Eq. (1.12) and define the quantity

$$\Gamma_{tot}(\underset{\sim}{b};\underset{\sim}{r}_1,\cdots,\underset{\sim}{r}_A) = 1 - e^{i\chi_{tot}(\underset{\sim}{b};\underset{\sim}{r}_1\cdots\underset{\sim}{r}_A)} \tag{1.16}$$

so that we obtain the formula

$$F_{fi}(\underset{\sim}{k}_i,\underset{\sim}{k}_f) = \frac{ik_i}{2\pi}\langle f|\int d^2\underset{\sim}{b}\; e^{i(\underset{\sim}{k}_i-\underset{\sim}{k}_f)\cdot\underset{\sim}{b}}\;\Gamma_{tot}(\underset{\sim}{b};\underset{\sim}{r}_1,\cdots,\underset{\sim}{r}_A)|i\rangle, \tag{1.17}$$

in close analogy with the two-body expression (1.6). Introducing the quantities

$$\Gamma_j(\underset{\sim}{b}-\underset{\sim}{b}_j) = 1 - e^{i\chi_j(\underset{\sim}{b}-\underset{\sim}{b}_j)}\;, \tag{1.18}$$

Glauber now writes

$$\Gamma_{tot} = 1 - \prod_{j=1}^{A}\left(1 - \Gamma_j(\underset{\sim}{b}-\underset{\sim}{b}_j)\right) \tag{1.19}$$

or

$$\Gamma_{tot} = \sum_j \Gamma_j - \sum_{j\neq\ell}\Gamma_j\,\Gamma_\ell + \cdots + (-)^{A-1}\prod_{j=1}^{A}\Gamma_j\;. \tag{1.20}$$

This last equation, when substituted into Eq. (1.17), leads directly to an interesting interpretation of the collision in terms of a multiple scattering expansion. The terms linear in Γ_j on the right-hand side of Eq. (1.20) account for the "single scattering" (impulse) approximation* to the scattering amplitude, whereas the next terms provide

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* It is worth pointing out that the terms "single scattering" and "impulse approximation" are sometimes defined differently in the literature.

double, triple···scattering corrections. We note that the order of multiple scattering can at most be A in high-energy diffraction theory, reflecting the fact that the scattering is focused in the forward direction.

Before discussing in detail the case of hadron-deuteron scattering, let us mention an important application of Eq. (1.12) to the coherent scattering of high-energy hadrons by nuclei (Stodolsky, 1966; Goldhaber and Joachain, 1968). In this case it is convenient to define an "optical" phase shift function $\chi_{opt}(\underset{\sim}{b})$ such that (Glauber, 1959)

$$e^{i\chi_{opt}(\underset{\sim}{b})} = \langle i|e^{i\sum_{j=1}^{A}\chi_j(\underset{\sim}{b}-\underset{\sim}{b}_j)}|i\rangle \tag{1.21}$$

and therefore--compare with Eq. (1.3)--

$$F_{ii} = \frac{ik_i}{2\pi}\int d^2\underset{\sim}{b}\; e^{i(\underset{\sim}{k}_i-\underset{\sim}{k}_f)\cdot\underset{\sim}{b}}\,[1 - e^{i\chi_{opt}(\underset{\sim}{b})}] \quad . \tag{1.22}$$

If we consider a sufficiently large nucleus for which the concept of nuclear density is meaningful, and assume that the target nucleons are uncorrelated, a simple expression may be obtained for χ_{opt} as

$$\chi_{opt}(\underset{\sim}{b}) = \lambda_i\, f_0 \int_{-\infty}^{+\infty} \rho(\underset{\sim}{b},z)\, dz, \tag{1.23}$$

where $\lambda_i = 2\pi\, k_i^{-1}$ is the de Broglie wavelength of the incident particle, f_0 is the forward hadron-nucleon scattering amplitude averaged over the spins and isospins of the target nucleons, and $\rho(\underset{\sim}{r})$

is the nuclear density normalized to A, the number of nucleons in the target. It is interesting to note that Eqs. (1.22) and (1.23) are identical to those obtained by computing first the optical potential in the "single scattering" approximation of Watson's multiple scattering formalism (Goldberger and Watson, 1964) and then "eikonalizing" the resulting potential. The formalism outlined here, together with additional corrections for Coulomb and target correlation effects, has been used by Goldhaber and Joachain (1968) to analyze the experimental data of Belletini et al. (1966) on high-energy hadron scattering by a variety of nuclei.

We shall not pursue further hadron scattering by nuclei other than deuterium. The interested reader will find additional information and references in recent work (Formánek and Trefil, 1967; Margolis, 1968; Kölbig and Margolis, 1968; Trefil, 1969; Feshbach and Hüfner, 1970) as well as in the review articles of Glauber (1967, 1969), Wilkin (1968), and Czyz (1970).

1.2. Hadron-deuteron Collisions

Let us now concentrate on hadron-deuteron collisions, which have been studied extensively by using Glauber's generalization of the eikonal approximation outlined above. We follow here the analysis of Franco and Glauber (1966). The basic formula for elastic and "mildly" inelastic collisions is Eq. (1.17), namely

$$F_{fi} = \frac{ik_i}{2\pi} \langle f | \int d^2\underset{\sim}{b} \; e^{i(\underset{\sim}{k}_i - \underset{\sim}{k}_f)\cdot \underset{\sim}{b}} \; \Gamma_{tot} | i \rangle \tag{1.24}$$

where

$$\Gamma_{tot} = 1 - e^{i[\chi_n(\underset{\sim}{b}-\frac{1}{2}\underset{\sim}{s})+\chi_p(\underset{\sim}{b}+\frac{1}{2}\underset{\sim}{s})]} . \tag{1.25}$$

The quantities χ_n and χ_p are the phase shifts contributed respectively by the neutron and the proton, while the vector $\underset{\sim}{s}$ is the projection of the internal relative vector $\underset{\sim}{r}_d$ of the deuteron in the plane of impact parameters. If we define the quantities

$$\Gamma_n(\underset{\sim}{b}) = 1 - e^{i\chi_n(\underset{\sim}{b})} \tag{1.26}$$

and

$$\Gamma_p(\underset{\sim}{b}) = 1 - e^{i\chi_p(\underset{\sim}{b})} \tag{1.27}$$

we may write Eq. (1.25) as

$$\Gamma_{tot} = \Gamma_n(\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{s}) + \Gamma_p(\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{s}) - \Gamma_n(\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{s})\,\Gamma_p(\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{s}) \tag{1.28}$$

leading to the physical interpretation in terms of single and double scattering, as we expect from the discussion following Eq. (1.20). To analyze this situation in more detail, we note that the functions Γ_n and Γ_p can be expressed in terms of hadron-nucleon scattering amplitudes f_{Xn} and f_{Xp} by an approximate two-dimensional Fourier inversion. [See Eq. (1.3).] Thus

$$\Gamma_{Xn}(\underset{\sim}{b}) \simeq \frac{1}{2\pi i k_i} \int d^2\underset{\sim}{q}\; e^{-i\underset{\sim}{q}\cdot\underset{\sim}{b}}\; f_{Xn}(\underset{\sim}{q}) , \tag{1.29}$$

where $\underset{\sim}{q}$ is the momentum transfer vector introduced in Eq. (1.4). A similar formula holds for Γ_{Xp}. Returning to Eq. (1.24), we now have

$$F_{fi} = \langle f | \Big\{ e^{i\frac{1}{2}\underset{\sim}{q}\cdot\underset{\sim}{s}} f_n(\underset{\sim}{q}) + e^{-i\frac{1}{2}\underset{\sim}{q}\cdot\underset{\sim}{s}} f_p(\underset{\sim}{q})$$

$$+ \frac{i}{2\pi k_i} \int d^2\underset{\sim}{q}' \; e^{i\underset{\sim}{q}'\cdot\underset{\sim}{s}} f_n(\underset{\sim}{q}' + \tfrac{1}{2}\underset{\sim}{q}) \, f_p(-\underset{\sim}{q}' + \tfrac{1}{2}\underset{\sim}{q}) \Big\} | i \rangle, \tag{1.30}$$

a formula which clearly justifies the interpretation of the collision in terms of single and double scattering processes. The two types of diagrams which contribute to the scattering are shown in Fig. 1-1. Evidently, these diagrams do not, at this point, have any more content than the formula (1.30). We shall return to the analysis of diagrams later in dealing with analytic properties of scattering amplitudes.

Let us now apply the optical theorem

$$\sigma_{Xd}^{tot} = \frac{4\pi}{k_i} \operatorname{Im} F_{ii}, \tag{1.31}$$

using Eq. (1.30) for F_{ii}. This yields for the total hadron-deuteron cross section

$$\sigma_{Xd}^{tot} = \sigma_{Xn}^{tot} + \sigma_{Xp}^{tot} - \delta\sigma, \tag{1.32}$$

where $\delta\sigma$, the "cross section defect," is given by

$$\delta\sigma = -\frac{2}{k^2} \int \phi(\underset{\sim}{q}) \operatorname{Re}[f_{Xn}(\underset{\sim}{q}) \, f_{Xp}(-\underset{\sim}{q})] \, d^2\underset{\sim}{q} \, . \tag{1.33}$$

Here $\phi(\underset{\sim}{q})$ is the form factor of the deuteron ground state, namely

$$\phi(\underset{\sim}{q}) = \int e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}} |\psi(\underset{\sim}{r})|^2 \, d^3\underset{\sim}{r} \, , \tag{1.34}$$

where $\psi(\underset{\sim}{r})$ is the ground-state deuteron wave function. If the average neutron-proton interaction has much larger range than the hadron-nucleon interaction, one readily obtains the approximate formula

$$\delta\sigma \simeq -\frac{4\pi}{k^2} \mathrm{Re}[f_{Xn}(0)\, f_{Xp}(0)] \langle r_d^{-2} \rangle, \tag{1.35}$$

where $\langle r_d^{-2} \rangle$ is the inverse square of the neutron-proton distance averaged over the deuteron ground state. Further, if the amplitudes $f_{Xn}(0)$ and $f_{Xp}(0)$ are purely imaginary ("black nucleons"), one obtains the very simple result (Glauber, 1959)

$$\delta\sigma \simeq \sigma_{Xn}^{tot}\, \sigma_{Xp}^{tot}\, \langle r_d^{-2} \rangle . \tag{1.36}$$

A variety of angular distributions can be derived from Eq. (1.24). The elastic differential cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{e\ell} = |F_{ii}(\underset{\sim}{q})|^2 . \tag{1.37}$$

The total scattered intensity is obtained from

$$\left(\frac{d\sigma}{d\Omega}\right)_{sc} = \sum_f |F_{fi}(\underset{\sim}{q})|^2, \tag{1.38}$$

and can be evaluated by using the closure relation on the deuteron final states. Inelastic processes in which the deuteron is dissociated into two free nucleons are calculated from

$$\left(\frac{d\sigma}{d\Omega}\right)_{in} = \left(\frac{d\sigma}{d\Omega}\right)_{sc} - \left(\frac{d\sigma}{d\Omega}\right)_{e\ell} . \tag{1.39}$$

The corresponding total cross sections $\sigma_{e\ell}$, σ_{sc}, and $\sigma_{in} = \sigma_{sc} - \sigma_{e\ell}$ are directly obtained by integrating Eqs. (1.37)-(1.39) over the angles, while the "absorption" cross section

$$\sigma_{abs} = \sigma_{Xd}^{tot} - \sigma_{sc} \tag{1.40}$$

corresponds to all processes where the incident hadron disappears during the collision or reappears with one or several produced particles.

The generalization of these considerations to include the spin and isospin degrees of freedom of the incident particle and the target nucleons has been carried out by several authors (Franco and Glauber, 1966; Wilkin, 1966; Glauber and Franco, 1967; Alberi and Bertocchi, 1968, 1969b). For example, collision processes contributing to charge-exchange scattering by the deuteron in the case of an incident hadron of isotopic spin 1/2 are represented in Fig. 1-2, whereas in Fig. 1-3 the double charge-exchange process leading to no net transfer of charge is shown. This last effect, first pointed out by Wilkin (1966), is small relative to the other cross-section corrections. Indeed, if $f_c(\underset{\sim}{q})$ is the charge-exchange amplitude, one obtains now for the cross-section defect, instead of Eq. (1.33)(Glauber and Franco, 1967),

$$\delta\sigma = -\frac{2}{k^2} \mathrm{Re} \int \phi(\underset{\sim}{q}) \frac{1}{2}[f_{Xp}(\underset{\sim}{q}) f_{Xn}(-\underset{\sim}{q}) + f_{Xn}(\underset{\sim}{q}) f_{Xp}(-\underset{\sim}{q}) - f_c(\underset{\sim}{q}) f_c(-\underset{\sim}{q})] \, d^2\underset{\sim}{q} , \tag{1.41}$$

or

$$\delta\sigma = -\frac{2}{k^2}\,\mathrm{Re}\int \phi(\underset{\sim}{q})[2f_{Xn}(\underset{\sim}{q})\,f_{Xp}(\underset{\sim}{q}) - \frac{1}{2}\,f_{Xp}{}^2(\underset{\sim}{q}) - \frac{1}{2}\,f_{Xn}{}^2(\underset{\sim}{q})]\,d^2\underset{\sim}{q}\ . \tag{1.42}$$

If the hadron-nucleon force range is small compared with the average neutron-proton interaction, one may again approximate

$$\delta\sigma \simeq -\frac{4\pi}{k^2}\,\mathrm{Re}\left[f_{Xn}(0)\,f_{Xp}(0) - \frac{1}{2}[f_{Xn}(0) - f_{Xp}(0)]^2\right]\langle r_d{}^{-2}\rangle \tag{1.43}$$

which under the assumption of purely imaginary amplitudes $f_{Xn}(0)$ and $f_{Xp}(0)$ reduces to [compare with Eq. (1.36)]

$$\delta\sigma \simeq \frac{1}{4\pi}[\sigma_{Xn}^{tot}\,\sigma_{Xp}^{tot} - \frac{1}{2}(\sigma_{Xn}^{tot} - \sigma_{Xp}^{tot})^2]\langle r_d{}^{-2}\rangle\ . \tag{1.44}$$

Franco and Glauber (1966) have applied the theory outlined above to a detailed investigation of antiproton-deuteron collisions in the (lab) energy range 0.13 to 17.1 GeV, using various ground-state deuteron wave functions. They assume that at high energies the antiproton-nucleon amplitudes are such that

$$f_{\bar{p}n}(\underset{\sim}{q}) = f_{\bar{p}p}(\underset{\sim}{q}) \equiv f_{\bar{p}N}(\underset{\sim}{q}), \tag{1.45}$$

and can be parameterized as

$$f_{\bar{p}N} = i(k_i\sigma_{\bar{p}N}/4\pi)\,e^{-\frac{1}{2}\alpha^2\underset{\sim}{q}^2}\ . \tag{1.46}$$

Using as input the measured experimental data (Elioff et al., 1962; Galbraith et al., 1965; Czyzewski et al., 1965; Coombes et al., 1958; Armenteros et al., 1960; Foley et al., 1963; Ferbel et al., 1965) on

antiproton-proton collisions, they obtained total and absorption antiproton-deuteron cross sections in good agreement with experiment (Elioff et al., 1962; Galbraith et al., 1965; Chamberlain et al., 1957) and showing an appreciable double scattering effect (see Fig. 1-4). They also investigated spin-dependent effects and concluded that their influence on the cross-section defect should be small. Franco (1966) has also analyzed the antiproton-deuteron elastic angular distribution for small momentum transfers in the region of incident momenta between 2.78 and 10.9 GeV/c. In a subsequent work, Glauber and Franco (1967) studied the reaction

$$K^+ + d \rightarrow K^0 + p + p, \tag{1.47}$$

which, together with (K^+p) collisions, is used to extract information about the K^+n charge-exchange reaction (Butterworth et al., 1965)

$$K^+ + n \rightarrow K^0 + p \ . \tag{1.48}$$

They show that the effect of the charge-exchange correction on the values of the (pn), $(\bar{p}n)$, and (K^+n) total cross sections which are obtained indirectly through deuteron measurements is very small for incident hadron momenta above 2 GeV/c.

We now turn to a more detailed analysis of the <u>angular distribution</u> of <u>elastic</u> hadron-deuteron scattering. We start with proton-deuteron elastic scattering, which has been studied in the GeV range by various authors (Harrington, 1964; 1968 a,b; Franco, 1966, 1968; Franco and Coleman, 1966; Kujawski, Sachs, and Trefil, 1968; Franco and Glauber, 1969). To understand qualitatively the main

features of the angular distribution, let us return to Eq. (1.30), which for $f = i$ reduces to

$$F_{ii}(\underset{\sim}{q}) = f_{Xn}(\underset{\sim}{q})\,\phi(\tfrac{1}{2}\underset{\sim}{q}) + f_{Xp}(\underset{\sim}{q})\,\phi(-\tfrac{1}{2}\underset{\sim}{q})$$

$$+ \frac{i}{2\pi k}\int \phi(\underset{\sim}{q}')\, f_{Xn}(\tfrac{1}{2}\underset{\sim}{q} + \underset{\sim}{q}')\, f_{Xp}(\tfrac{1}{2}\underset{\sim}{q} - \underset{\sim}{q}')\, d^2\underset{\sim}{q}', \qquad (1.49)$$

where $\phi(\underset{\sim}{q})$ is the deuteron form factor defined in Eq. (1.34) and X is the incoming proton. We first note from the alternation of sign in Eq. (1.28) that the double scattering term has opposite sign to the single scattering term. In fact, if the amplitudes f_{Xn} and f_{Xp} were purely imaginary, as in Eq. (1.42), the double scattering term would completely cancel the contribution of the single scattering amplitude at $-t \simeq 0.5\ (\mathrm{GeV}/c)^2$. The contribution of the single and double scattering terms for such a parametrization of the amplitudes f_{XN} is displayed in Fig. 1-5, which also shows that the single scattering term dominates near the forward direction. At larger momentum transfers the double scattering term, which decreases much more slowly with increasing q, becomes the dominant contribution to the scattering amplitude.

Let us now analyze more closely the intermediate region of momentum transfers where the single and double scattering terms interfere destructively.* Since the proton-neutron and proton-proton scattering amplitudes both have small real parts we do not expect the differential cross section to exhibit a zero, but instead to show a sharp dip in the interference region. This region is therefore of

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* This was first noticed by Franco and Glauber (1966).

special interest, since it depends delicately upon the phases of the hadron-nucleon amplitudes.

The first experimental data on p-d elastic scattering (Kirillova et al., 1964; Belletini et al., 1965; Zolin et al., 1966; Coleman et al., 1966, 1967) gave encouraging agreement with Glauber's theory. For example, the large-angle measurements at 2.0 GeV/c (Coleman et al., 1966) confirmed the importance of the double scattering term in the region of four-momentum transfers

$$0.5\ (\mathrm{GeV/c})^2 \leq -t \leq 1.5\ (\mathrm{GeV/c})^2, \tag{1.50}$$

and were in excellent agreement with the theoretical calculations of Franco and Coleman (1966). However, these larger-angle data did not fully cover the important intermediate region. It remained for Bennett et al. (1967) to perform a crucial p d experiment at 1 GeV, which showed agreement with the theory in the small and larger momentum transfer ranges, but displayed only a shoulder (no dip) in the interference region (see Fig. 1-6). This result was confirmed by measurements at 582 MeV (Boschitz, quoted in Glauber 1969). A similar feature was observed in π^-d elastic scattering experiments (Bradamante et al., 1968).

Several suggestions were proposed to understand this apparent paradox: momentum-transfer dependence of the phases of the proton-neutron and proton-proton amplitudes (Bennett et al., 1967), spin effects (Kujawski, Sachs, and Trefil, 1968; Franco, 1968), influence of three-body forces (Harrington, 1968a) or of inelastic intermediate states (Pumplin and Ross, 1968; Alberi and Bertocchi, 1969a; Harrington, 1970). As in any good thriller, though, there is one crucial fact which

leads to the solution of the difficulty, namely that interference minima are observed in the elastic scattering of protons by the spin-zero nuclei He^4, C^{12}, and O^{16}. (Palevsky et al., 1967; Boschitz et al. 1968) It is therefore tempting to associate the absence of the dip with the quadrupole deformation of the deuteron, because this nucleus has spin one (Harrington, 1968a). Detailed calculations taking into account the D-state of the deuteron have been done for (p,d) scattering by Franco and Glauber (1969) and are in good agreement with the experimental data. Figure 1-7 shows the comparison of the theoretical calculations (Franco and Glauber, 1969) and the experimental data at 1 and 2 GeV. Recent elastic p d experiments at 12.8 GeV/c (Bradamante et al., 1969b; Fidecaro et al., quoted in Glauber, 1969) and at higher energies (Allaby et al., 1969a,b) are also in excellent agreement with the improved version of the theory.

Similar considerations apply to pion-deuteron elastic scattering. Alberi and Bertocchi (1969b) have re-analyzed the data of Bradamante et al. (1968) by taking into account the D state of the deuteron. Their results are shown in Fig. 1-8, which exhibits impressive agreement between theory and experiment. Analogous calculations by Michael and Wilkin (1969) also agree very well with the $\pi^- d$ elastic differential cross sections of Fellinger et al. (1969) and Bradamante et al. (1968, 1969a, 1970) for incident pion momenta between 2 and 15.2 GeV/c (Fig. 1-9).

Since the scattering amplitude for elastic hadron-deuteron scattering in the intermediate momentum transfer region is dominated by quadrupole transitions between the deuteron S and D states, it is

strongly dependent on the relative orientations of the momentum transfer and the deuteron spin. Thus, as Franco and Glauber (1969) have pointed out, interesting effects could appear in experiments involving polarized deuteron targets. Indeed, with such a target, the interference dip can appear or not depending on the particular experimental arrangement, namely on the orientation of the polarization axis. Another interesting experiment using the spin-dependence arising from the D-wave component of the deuteron to produce high-energy aligned deuterons has been proposed by Harrington (1969a). It consists in letting high-energy incident deuterons collide with protons, so that the scattered deuterons will be strongly polarized.

Finally let us mention the experiments of Carter et al. (1968), who measured π-d cross sections, and of Chase et al. (1969) on inelastic pion-deuteron scattering at 5.53 GeV/c, leading to an outgoing pion plus anything in the final state (missing-mass experiment). The inelastic intensity, calculated from Eq. (1.39), was found to be in good agreement with the data. See also Hsiung et al. (1968).

1.3. Extensions of the Glauber Method

The Glauber formalism which we have reviewed so far leads to remarkably simple formulae which can easily be interpreted physically. Let us recapitulate the basic ingredients of Glauber's method:

(1) Use of the eikonal approximation valid for high-energy and small angles and neglect of the longitudinal momentum transfer.

(2) Neglect of the "reaction" of target nucleons.

(3) Phase additivity, i.e., the total phase-shift function is the sum of the phase shifts for each individual hadron-nucleon scattering.

(4) Neglect of three-body forces.

As we have emphasized above, this method is at its best for collisions in which the inelasticity is small, in particular for elastic scattering, for which the results in the high-energy small-angle limits are in excellent agreement with the data. Even in that case, however, one should keep in mind that several correction terms, typified by the contribution of inelastic intermediate states (see Fig. 1-10) should be included in the scattering amplitude. There is no simple way to take into account the contribution of such inelastic intermediate states within the framework of Glauber's method. Fortunately, because of the mass difference, the reaction

$$p + p \rightarrow p + N^* \tag{1.51}$$

has a minimum momentum transfer greater than zero, so that two relatively large angle scatterings of this type, leaving the deuteron in its bound state, are not likely to occur with high probability compared with the single and double scattering terms discussed before. Such "truly inelastic" corrections have been considered for proton-nucleon scattering by Pumplin and Ross (1968) and for pion-deuteron scattering by Alberi and Bertocchi (1969a) and Harrington (1970).

The excellent agreement between conventional Glauber theory and the 19.1 GeV/c pd data of Allaby, et al. (1969) indicates that inelastic corrections are negligible at that momentum.

More serious problems arise when one wants to study coherent production reactions such as

$$\pi + d \rightarrow A_1 + d \tag{1.52}$$

or

$$p + He^4 \rightarrow N^* + He^4, \text{ etc.} \tag{1.53}$$

Goldhaber and Joachain (1968) have proposed a simple, DWBA-type method to deal with such reactions when the target nucleus is "large." This formalism has been applied to extract the A_1-nucleon cross section from the analysis of coherent A_1 production in freon (Goldhaber, Joachain, Lubatti, and Veillet, 1969). Similar ideas could possibly be applied to analyze reactions like (1.52)-(1.53).

A more ambitious attempt at improving Glauber's method is to proceed in a systematic way from a more general formalism. Leaving aside the methods based on analytic properties of the scattering amplitudes, to which we shall return later, we start here from the multiple scattering formalism (Goldberger and Watson, 1964; Kerman, McManus, and Thaler, 1959). Analyses along these lines have been made recently by Czyz and Maximon (1968, 1969) and by Remler (1968). Since we are particularly interested in hadron-deuteron scattering, it is convenient to start from the Faddeev equations (Faddeev 1960, 1961, 1962.) which correctly describe three-body collisions, at least in the nonrelativistic limit.*

Let us label the incoming hadron as particle 1 and the proton and neutron of the deuteron as particles 2 and 3, respectively. The total scattering operator can be written as

$$T = T^{(1)} + T^{(2)} + T^{(3)}, \tag{1.54}$$

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* Generalizations of the Faddeev equations to include relativistic kinematics have been proposed by several authors (Freedman, Lovelace, and Namylowski, 1966; Alessandrini and Omnès, 1965; Blankenbecler and Sugar, 1966).

where $T^{(1)}$ is the sum of all diagrams which contribute to T in which particles 2 and 3 interact last. The Faddeev equations then read

$$\begin{pmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + \begin{pmatrix} 0 & T_1 & T_1 \\ T_2 & 0 & T_2 \\ T_3 & T_3 & 0 \end{pmatrix} G_0 \begin{pmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{pmatrix} \tag{1.55}$$

where G_0 is the free Green's function of the system and T_i $(i=1,2,3)$ are the two-body T matrices corresponding to two-particle scattering with the third particle i free. It should be noted that the Faddeev equations are essentially equivalent to Watson's multiple scattering equations (Goldberger and Watson, 1964; Watson and Nuttall, 1967). Application of the appropriate boundary conditions then leads to multiple scattering expressions for various three-body collision processes. For example, in the case of elastic scattering, we have

$$T = T_2 + T_3 + T_2 G_0 T_3 + T_3 G_0 T_2 + \cdots \quad . \tag{1.56}$$

Bhasin (1967) has made a detailed study of this equation. As expected, the two first terms on the right reduce to Glauber's single scattering terms if one ignores the dependence of T_2 or T_3 on the energy of the third particle and also assumes the two-body off-the-energy-shell amplitudes to be functions only of the momentum transfer. With these assumptions and the additional requirement that $\underset{\sim}{k}_i \simeq \underset{\sim}{k}_f$, the double scattering terms $T_2 G_0 T_3$ and $T_3 G_0 T_2$ also reduce to the Glauber "eclipse" correction. Pumplin (1968) and

Bhasin and Varma (1969) have investigated the importance of the off-shell corrections on the double scattering terms. They find that the corresponding effect for proton-deuteron scattering is largest in the interference region between single and double scattering. However, Harrington (1969b) has recently shown that in a potential model the off-energy-shell effects in the double scattering term must cancel the contribution of the remaining part of the multiple scattering series in the high-energy limit.* It should be noted here that only in high-energy diffraction theory does the multiple scattering series terminate after A terms. In the deuteron case considered here the triple, quadruple,··· terms are small, since they contain at least one (unlikely) backward scattering. Their sum could well annihilate the off-energy-shell contribution to the double scattering term, if the mechanism described by Harrington also works for interactions which cannot be described by potentials.

While we are still discussing the multiple scattering series, it is worth mentioning a recent paper by Kofoed-Hansen (1969), who has pointed out that truncated versions of the Glauber series (1.20) could produce misleading results, since the series is slowly converging in terms of multiplicity. This remark, evidently, does not apply to the deuteron case--where the multiplicity is two--but is relevant in cases such as nucleus-nucleus collisions (Franco, 1967, 1970) as well as in quark model or multiple scattering theories of hadron-hadron scattering (Harrington and Pagnamenta, 1967, 1968, 1969; Deloff, 1967;

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* Harrington's calculation will be discussed in more detail in Sec. 2.1.

Barnhill, 1967; Schrauner, Benofy, and Cho, 1967; Chou and Yang, 1968; Frautschi and Margolis, 1968; Durand and Lipes, 1968).

We now return to the question of relaxing some of the basic assumptions of Glauber's theory. A simple approach to the problem of extending the eikonal method to scattering angles which, although geometrically small ($\theta \ll 1$) could be dynamically large, $\theta \simeq (kd)^{-\frac{1}{2}}$, where d is the characteristic "eikonal distance" (Glauber, 1959) has been recently suggested by Schiff (1968). The procedure followed by Schiff consists in interpolating between two expressions, valid respectively in the small and large angle limit. (Schiff, 1956; Saxon and Schiff, 1957). He writes the scattering amplitude as

$$f(\underset{\sim}{q}) = \frac{k}{2\pi} \int d^2\underset{\sim}{b} \int_{-\infty}^{+\infty} dz\, \kappa(\underset{\sim}{b},z)\, e^{i[\underset{\sim}{q}\cdot\underset{\sim}{b} + \int_{-\infty}^{z} \kappa(\underset{\sim}{b},z')dz' + \gamma(\theta)\int_{z}^{\infty} \kappa(\underset{\sim}{b},z')dz']} \tag{1.57}$$

where $\kappa(\underset{\sim}{r})$ is the wave-number shift and $\gamma(\theta)$ is some unknown function of θ, such that

$$\begin{aligned} \gamma(\theta) &= 0 \quad \text{in the small-angle limit} \\ \gamma(\theta) &= 1 \quad \text{in the large-angle limit.} \end{aligned} \tag{1.58}$$

Performing the z integration in Eq. (1.57) leads to

$$f(\underset{\sim}{q}) = \frac{ik}{2\pi[1 - \gamma(\theta)]} \int d^2\underset{\sim}{b}\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{b}} \left[e^{i\gamma(\theta)\chi(\underset{\sim}{b})} - e^{i\chi(\underset{\sim}{b})}\right] \tag{1.59}$$

where

$$\chi(\underset{\sim}{b}) = \int_{-\infty}^{+\infty} \kappa(\underset{\sim}{b},z')\, dz' \quad . \tag{1.60}$$

Clearly, the Eq. (1.55) reduces to the eikonal expression (1.3) for $\gamma(\theta) = 0$ and leads to

$$f(\underset{\sim}{q}) = \frac{k}{2\pi} \int d^2\underset{\sim}{b}\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{b}}\, \chi(\underset{\sim}{b})\, e^{i\chi(\underset{\sim}{b})} \tag{1.61}$$

in the large-angle limit. It is interesting to note that Eq. (1.61) has been proposed in another context by Byers and Yang (1966). Ross (1968) has used this method to study proton-He^4 scattering at larger angles and compare it with the Glauber-type calculations of Bassel and Wilkin (1967, 1968), and Czyz and Leśniak (1967). An important drawback of Schiff's method, however, is that $\gamma(\theta)$ is an arbitrary, possibly complex function subject only to the instruction (1.58), so that too much freedom is left in the parametrization of it.

Finally, let us consider briefly the effect of three-body forces in hadron-deuteron collisions. Harrington (1968b) has studied corrections to the Glauber expansion due to the scattering of the incident hadron from a pion being exchanged by the two target nucleons. Numerical estimates indicate that such an effect on the total cross section is quite small (<1% at very high energies), but could possibly influence the differential cross section at large momentum transfers.

2. Hadron-Deuteron Scattering and Regge Theory

2.1. Glauber Theory in the J Plane

How to calculate Regge cuts (branch cuts in the angular momentum plane) is one of the challenging theoretical problems of the present day, for which no solution seems close at hand. We therefore choose an historical approach to the relation between the Glauber formalism and Regge theory. In this way we shall encounter some of the false steps which have been taken in the past, and try to convey the theoretical atmosphere of the present. Some insight is gained into the connection between diffraction and Regge poles if, following Udgaonkar and Gell-Mann (1962), we understand the shrinkage of the diffraction peak by an optical analogue.

At high energies hadron-hadron scattering is apparently dominated by Pomeranchuk exchange. The X-Y elastic scattering invariant amplitude, which we represent in Fig. 2-1, has the form for small angles

$$A_{XY}(s,t) = \{i - \cot[\pi\alpha_P(t)/2]\}\ \gamma_X(t)\ \gamma_Y(t)\ s_0\left(\frac{s}{s_0}\right)^{\alpha_P(t)} , \qquad (2.1)$$

where $s = -(p_X' + p_Y)^2$ is the square of the total c.m. energy, $t = -(p_X - p_X')^2$ is the square of the four-momentum transfer, s_0 is the Regge scale energy-squared, and $\alpha_P(t)$ is the Pomeranchuk trajectory function: $\alpha_P(0) \doteq 1$. The total cross section is given in terms of this amplitude by the optical theorem,

$$\sigma_{total}(s) \approx \frac{1}{s}\,\mathrm{Im}[A_{XY}(s,0)] = \gamma_X(0)\ \gamma_Y(0) \ . \qquad (2.2)$$

Here we have explicitly exposed the factorization property of the pole residues. Let us rewrite (2.1) as

$$A(s,t) = \{i - \cot[\pi\alpha_P(t)/2]\}\, s_0\left(\frac{s}{s_0}\right)^{\alpha_P(t)} \sigma_{total}(s)$$

$$\cdot\, [\gamma_X(t)\, \gamma_Y(t)/\gamma_X(0)\, \gamma_Y(0)] \quad . \tag{2.3}$$

Now assume that the Pomeranchuk trajectory is linear, $\alpha_P(t) = 1 + \epsilon t$, and that the residue functions are slowly varying, so we may set the factor in square brackets equal to 1. Then for small t, we have

$$A(s,t) \approx i\, s\, \sigma_{total}(s)\, e^{\epsilon t \log\left(\frac{s}{s_0}\right)} \quad , \tag{2.4}$$

which exhibits, for $\epsilon > 0$, the shrinkage of the diffraction peak.

We write the partial-wave series for $A(s,t)$,

$$A(s,t) = 8\pi i \sum_{\ell} (2\ell + 1)\, P_\ell(\cos\theta_s)(1 - e^{2i\delta_\ell}) \quad . \tag{2.5}$$

We turn the sum over ℓ into an integral, introduce the impact parameter $b = 2\ell\, s^{-\frac{1}{2}}$, and use $P_\ell(\cos\theta_s) \approx P_\ell(1 + 2t/s) \approx J_0[b(-t)^{\frac{1}{2}}]$. We then calculate $f(s,t)$, an amplitude such that $\frac{d\sigma}{dt} = |f(s,t)|^2$, which for NN scattering at high energies is $f(s,t) \approx [4s(\pi)^{\frac{1}{2}}]^{-1} A(s,t)$, so that

$$\left.\begin{aligned} f(s,t) &= \frac{i}{2(\pi)^{\frac{1}{2}}} \int_0^\infty 2\pi b\, db\, [1 - S(b,s)]\, J_0[b(-t)^{\frac{1}{2}}] \\ &= \frac{i}{2(\pi)^{\frac{1}{2}}} \int d^2\underset{\sim}{b}\, [1 - S(\underset{\sim}{b},s)]\, e^{i\underset{\sim}{b}\cdot\underset{\approx}{q}} \end{aligned}\right\} , \tag{2.6}$$

where the transmission coefficient $S(\underset{\sim}{b},s) = e^{2i\delta_\ell}$ and $q^2 = -t$. In the exponential approximation (2.4) Fourier inversion gives the absorption coefficient [now $\sigma \equiv \sigma_{total}(s)$]

$$1 - S(\underset{\sim}{b},s) \approx \frac{\sigma}{8\pi}[\epsilon \log(\frac{s}{s_0})]^{-1} \exp\{-b^2/[4\epsilon \log(\frac{s}{s_0})]\}. \tag{2.7}$$

Evidently the effective radius-squared (the value of b^2 for which the absorption coefficient is $1/e$ times its value at $b = 0$) is $4\epsilon \log(\frac{s}{s_0})$, which increases logarithmically with s. Likewise the transparency, which we define as $[1 - S(b = 0,s)]^{-1}$, is logarithmically increasing with s because of the factor $\epsilon \log(\frac{s}{s_0})$. Finally we find that the elastic cross section,

$$\sigma_{e\ell} = \int d^2\underset{\sim}{b}|1 - S(\underset{\sim}{b},s)|^2 \approx \frac{\sigma^2}{32\pi\ \epsilon \log(\frac{s}{s_0})}, \tag{2.8}$$

tends to zero as $s \to \infty$.

Let us describe a nucleus approximately as a composite system specified by a wave function referring to the individual coordinates of the constituent nucleons. Assuming that high-energy NN scattering is controlled by Regge poles, we compute the amplitude for high-energy N-d scattering. The probability distribution $|\psi|^2$ of the nucleon positions is integrated over the beam direction (z coordinates) to give a probability distribution $P(\underset{\sim}{b}_1,\underset{\sim}{b}_2)$ of two-dimensional vectors $\underset{\sim}{b}_i$. Then the transmission coefficient for the deuteron, $S_d(\underset{\sim}{b},s)$, is just the averaged product of the transmission coefficients for the constituent nucleons:

$$S_d(\underset{\sim}{b},s) = \int d^2\underset{\sim}{b}_1 \int d^2\underset{\sim}{b}_2\, P(\underset{\sim}{b}_1,\underset{\sim}{b}_2)\, S(\underset{\sim}{b} - \underset{\sim}{b}_1,s)\, S(\underset{\sim}{b} - \underset{\sim}{b}_2,s). \tag{2.9}$$

Now we take the deuteron c.m. as the origin so that $\underset{\sim}{b}_1 = \frac{1}{2}\underset{\sim}{\rho}$ and $\underset{\sim}{b}_2 = -\frac{1}{2}\underset{\sim}{\rho}$, where $\underset{\sim}{\rho}$ is the two-dimensional relative coordinate. Let the wave function--ignoring spin-- be $\psi(\underset{\sim}{\rho},z)$ and define

$$G(p^2) = \int_{-\infty}^{\infty} dz \int d^2\underset{\sim}{\rho}\,|\psi(\underset{\sim}{\rho},z)|^2\, e^{i\underset{\sim}{p}\cdot\underset{\sim}{\rho}}\,. \tag{2.10}$$

Then we get for the scattering amplitude and total cross section

$$f_{Xd}(s,t) = \frac{i}{4(\pi)^{\frac{1}{2}}}\Bigg\{2\sigma\, G(-t/4)\, B(t)\left(\frac{s}{s_0}\right)^{\alpha(t)-1}$$

$$- \frac{\sigma^2}{8\pi^2}\int d^2\underset{\sim}{p}\, G(p^2)\, B\left(-\left(\frac{\underset{\sim}{q}}{2} - \underset{\sim}{p}\right)^2\right) B\left(-\left(\frac{\underset{\sim}{q}}{2} + \underset{\sim}{p}\right)^2\right)$$

$$\times \left(\frac{s}{s_0}\right)^{\alpha\left(-\left(\frac{\underset{\sim}{q}}{2}+\underset{\sim}{p}\right)^2\right)+\alpha\left(-\left(\frac{\underset{\sim}{q}}{2}-\underset{\sim}{p}\right)^2\right)-2}\Bigg\} \tag{2.11}$$

and

$$\sigma_{Xd}^{tot} = 2\sigma - \frac{\sigma^2}{8\pi^2}\,\mathrm{Re}\int d^2\underset{\sim}{p}\, G(p^2)\,[B(-p^2)]^2\left(\frac{s}{s_0}\right)^{2\alpha(-p^2)-2}, \tag{2.12}$$

where $B(t) = \{1 + i\cot[\pi\alpha(t)/2]\}\, s_0\, \gamma_X(t)\, \gamma_Y(t)/\gamma_X(0)\, \gamma_Y(0)$ is the Regge residue function.

In addition to the Pomeranchuk pole term, with a coefficient twice as large in the forward direction as in the NN case, there is an eclipse term which corresponds to a continuous "smear" of Regge poles, i.e. to a Regge cut with branch point at

$$\alpha_c = 2\alpha(t/4) - 1. \tag{2.13}$$

This is the result of Udgaonkar and Gell-Mann (1962). At very high energies, the eclipse term at $t = 0$ vanishes like $1/\log(s/s_0)$ and $\sigma_{Xd}^{tot} \to 2\sigma$. This is sensible because, as we saw above, the nucleons become very transparent at high energies. For intermediate energies, the eclipse term can be identified with Glauber's.

Abers et al. (1966) observed that from the point of view of Feynman graphs the double scattering term contains no Regge cut, so the validity of the result of Udgaonkar and Gell-Mann and, by extension, of Glauber theory at high energies is questionable. To compress this discussion somewhat we draw from a recent lecture by Wilkin (1969). We may represent the Glauber terms graphically as the impuse (or single scattering) terms of Figs. 2-2a,b and the eclipse (or double scattering) term of Fig. 2-2c. Regarded as a Feynman diagram, the double scattering graph has no Regge cut, because the off-mass-shell part of the loop integral cancels the Regge cut from the on-mass-shell contribution which is obtained by replacing the propagator by a delta function. Thus it contributes asymptotically only as s^{-3}, not as $s/\log s$, which is given by the Glauber formula. A general Feynman diagram as in Fig. 2-3 has a j-plane branch cut on the physical sheet only if both the left hand and the right-hand blobs have nonzero third double spectral functions $[\rho_{su}(s,t)]$ in the t-channel sense. In other words, crossed lines are required on both sides of the graph; the simplest diagram with a Regge cut appears in Fig. 2-4a [cf. Mandelstam (1963), Wilkin (1964)]. Such a result must be a source of embarrassment either for

Glauber theory as embodied in the calculation of Udgaonkar and Gell-Mann or for Feynman graphs, if not for both. On the one hand Feynman diagrams are "fundamental" and therefore to be believed. On the other, Glauber theory has been checked experimentally for energies up to a few GeV.

One may try to circumvent the difficulty by imputing to the projectile hadron an internal structure which includes a cross (e.g., Fig. 2-4b) and claiming that the compositeness of hadrons restores the Regge cut. Such a calculation was performed by Abers et al. (1966), who thereby proposed to replace the Glauber eclipse term with a complicated expression dependent upon the internal structure of the projectile. Assigning a particular internal structure to the projectile seems artificial, especially when the imputed structure may be absent. As Quigg (1970) emphasizes, the statement $\rho_{su} \neq 0$ is equivalent to the statement that the projectile has definite (s-channel) signature. To the extent that exchange degeneracy is exact hadrons do not have definite signature and the cross, artificial or not, simply does not correspond to physics. This phenomenological argument provides strong circumstantial evidence against the imputed structure Feynman graph approach.

Landshoff (1969) has estimated the energy at which the Glauber theory result (the Regge cut of Udgaonkar and Gell-Mann) ceases to be valid numerically under the assumption that the relevant amplitude is given by the Feynman graph of Fig. 2-2c, without assigning any structure to the projectile. As the deuteron is very lightly bound, the critical laboratory energy at which the Glauber theory should break down is very large,

$$E_{critical} \approx m_{projectile}(M_{nucleon}/\text{Deuteron Binding Energy})^{\frac{1}{2}}. \tag{2.14}$$

For incident nucleons this is about 20 GeV. Thus while the Feynman diagram considered has no cut in the j plane its numerical properties are quite similar over a wide range of energy to those of the Glauber eclipse term.

Further doubt has been cast upon the simple diagram approach by a potential theory calculation of Harrington (1969b). In Glauber theory the amplitude for scattering from a potential V is

$$f(\underset{\sim}{k}_i,\underset{\sim}{k}_f) = \frac{k_i}{2\pi i}\int d^2\underset{\sim}{b}\; e^{i(\underset{\sim}{k}_i-\underset{\sim}{k}_f)\cdot\underset{\sim}{b}}\,[e^{i\chi(\underset{\sim}{b})} - 1], \tag{1.3}$$

where

$$\chi(\underset{\sim}{b}) = -\frac{1}{v_i}\int_{-\infty}^{\infty} dz\; V(\underset{\sim}{b},z). \tag{1.7}$$

We let $\underset{\sim}{q} \equiv \underset{\sim}{k}_i - \underset{\sim}{k}_f$ and invert the Fourier integral (1.3). Thus with $f(\underset{\sim}{q}) \equiv f(\underset{\sim}{k}_i,\underset{\sim}{k}_f)$ we get

$$\tilde{f}(\underset{\sim}{b}) = \frac{1}{2\pi}\int d^2\underset{\sim}{q}\; e^{-i\underset{\sim}{q}\cdot\underset{\sim}{b}}\, f(\underset{\sim}{q}). \tag{2.15}$$

In momentum space we have

$$\tilde{V}(\underset{\sim}{p}) = \frac{1}{(2\pi)^3}\int d^3\underset{\sim}{X}\; e^{i\underset{\sim}{p}\cdot\underset{\sim}{X}}\, V(\underset{\sim}{X}), \tag{2.16}$$

and the phase shift expressed in terms of $\tilde{V}$ is

$$\chi(\underset{\sim}{b}) = -\frac{2\pi}{v_i}\int d^2\underset{\sim}{q}\; e^{-i\underset{\sim}{q}\cdot\underset{\sim}{b}}\,\tilde{V}(\underset{\sim}{q}). \tag{2.17}$$

We expand the integrand of (1.3) in powers of $i\chi(\underset{\sim}{b})$,

$$f = \frac{k_i}{2\pi i}\int d^2\underset{\sim}{b}\; e^{i\underset{\sim}{q}\cdot\underset{\sim}{b}}\sum_{n=1}^{\infty}\frac{[i\chi(\underset{\sim}{b})]^n}{n!}, \tag{2.18}$$

and substitute (2.17) into (2.18) to obtain

$$f = -2\pi i\, k_i\sum_{n=1}^{\infty}\frac{1}{n!}\int d^2\underset{\sim}{q}_1\cdots\int d^2\underset{\sim}{q}_{n-1}\left(\frac{-2\pi i}{v_i}\right)^n$$

$$\times\;\tilde{V}(\underset{\sim}{q}_1)\,\tilde{V}(\underset{\sim}{q}_2)\cdots\tilde{V}(\underset{\sim}{q}_{n-1})\,\tilde{V}(\underset{\sim}{q}-\sum_{i=1}^{n-1}\underset{\sim}{q}_i). \tag{2.19}$$

This represents an infinite sum of ladder graphs in which the Feynman loop integrals are integrated only over transverse momentum components. We can reexpress (1.3) in terms of the Born amplitude

$$f_B(\underset{\sim}{q}) = -2\pi^2\,\tilde{V}(\underset{\sim}{q}), \tag{2.20}$$

$$f = -2\pi i\, k_i\sum_{n=1}^{\infty}\frac{1}{n!}\int d^2\underset{\sim}{q}_1\cdots\int d^2\underset{\sim}{q}_{n-1}$$

$$\times\left(\frac{if_B(\underset{\sim}{q}_1)}{v_i\pi}\right)\cdots\left(\frac{if_B(q-\sum_{i=1}^{n-1}\underset{\sim}{q}_i)}{v_i\pi}\right). \tag{2.21}$$

Thus we have a prescription for calculating the "absorptive corrections" to any Born term f_B. Wilkin next applies these rules to πd scattering to give some intuitive background to Harrington's result. First

notice that the vertex $d \to np$ is merely a deuteron wave function which we write in momentum space as $\phi(\underset{\sim}{p})$. If the πp amplitude of Fig. 2-2b is the Born term $f_B^p(\underset{\sim}{q})$ we get

$$\int d^3 \underset{\sim}{q}' \; \phi(\underset{\sim}{q}/4 - \underset{\sim}{q}') \; \phi(\underset{\sim}{q}/4 + \underset{\sim}{q}') \; f_B(\underset{\sim}{q}), \tag{2.22}$$

which is the expected result. It is straightforward to verify that the right answer is obtained for Fig. 2-2c.

Now consider the graphs in Fig. 2-5. Remarkably, both of these give the same answer,

$$\int d^3\underset{\sim}{p} \int d^2\underset{\sim}{q}_1 \; \phi(\underset{\sim}{p}) \; \phi(\underset{\sim}{p} + \underset{\sim}{q}_1) \; f_B^{\,n}(\underset{\sim}{q}_1 - \underset{\sim}{q}/2)$$

$$\times \int d^2\underset{\sim}{q}_2 \; f_B^{\,p}(\underset{\sim}{q}_2) \; f_B^{\,p}(\underset{\sim}{q}/2 + \underset{\sim}{q}_1 - \underset{\sim}{q}_2) \; , \tag{2.23}$$

which is recognizable as part of the Glauber multiple scattering term expanded in a Born series. Thus the Glauber theory includes <u>triple</u> scattering terms such as those in Fig. 2-5. Notice that the ordering of the πp and πn potential interactions does not affect the contribution of the graph. This is true for any complicated graph, as can be proved from the rules obtained above. It is then a basic property of Glauber theory that the order in which the interactions take place does not matter. A picturesque explanation of this fact [Wilkin (1969)] is that in deriving Glauber theory it is always assumed that the incident energy is large and any changes are very small. Complementary to this certainty in energy is an uncertainty

in time: it is impossible to tell which interaction takes place first and hence there is a commutativity among the several scatterings. Glauber theory exploits this independence of time order by lumping all the πp interactions together at one end of the (Glauber, not Feynman!) diagram and pushing all the πn interactions to the other end.

Harrington's calculation goes further. Employing the Faddeev multiple scattering series (cf. Sec. 1.3 of this review) he proves that in the high energy limit and in the Glauber approximation the off-shell contribution to the double scattering term is canceled by the higher order terms in the series. The proof consists in observing that in the high-energy limit the scattering amplitude is given by the Glauber approximation

$$T \sim T_{\text{Glauber}} = \sum_n T^{(n)}_{\text{Glauber}}, \tag{2.24}$$

where $T^{(n)}_{\text{Glauber}}$ is $T^{(n)}$ after the Glauber approximations have been made. If we break the linearized propagator into its δ-function $[\delta]$ and principal value $[P]$ (off-mass-shell) parts and correspondingly separate $T^{(2)}_{\text{Glauber}}$ as

$$T^{(2)}_{\text{Glauber}} = T^{(2)}_{\text{Glauber},\delta} + T^{(2)}_{\text{Glauber},P}, \tag{2.25}$$

then

$$T_{\text{Glauber}} = T^{(1)}_{\text{Glauber}} + T^{(2)}_{\text{Glauber},\delta}. \tag{2.26}$$

Thereby it follows that in the high-energy limit the off-shell contribution to $T^{(2)}$ must be canceled by the higher-order terms in the multiple scattering series

$$T_P^{(2)} + \sum_{n=3}^{\infty} T^{(n)} \sim T^{(2)}_{\text{Glauber},P} + \sum_{n=3}^{\infty} T^{(n)}_{\text{Glauber}} = 0. \qquad (2.27)$$

It is not known whether this exact cancellation carries over to the relativistic domain, but the likelihood that more complicated diagrams will continue to be important means that the use of a few Feynman graphs to debunk (or derive!) Glauber is a very dubious procedure. There is a lesson here for Regge cut calculations in nonnuclear hadron-hadron scattering as well. [We do not pursue non-deuteron scattering any further here, but recommend a sampling of opinion. For the connection between multiple scattering and Regge cuts see the discussion by Jackson (1970). The case against imputed structure is developed at more length by Quigg (1970), but for an opposing view, the rejection of exchange degeneracy, see Risk (1970).]

2.2. Singularities in the Mandelstam Plane

We shall not dwell on the analytic structure of the hadron-deuteron scattering amplitude in the momentum variables, for we are able to refer the reader to the elegant review by Ericson and Locher (1969) on hadron-nucleus forward dispersion relations. In the language of S-matrix theory, the lightly bound structure of the deuteron is evidenced through the existence of anomalous threshold singularities (so called because they cannot be discerned in straightforward fashion from unitarity) in $d \to ab$ Regge residue functions [Karplus et al. (1958)]. A rather complete discussion of the singularities of the dpn Regge residue function has recently been given by Lee (1968). Here we

content ourselves with recalling for the reader what anomalous singularities are, by giving an intuitive discussion due to Bohr (1960).

Consider the virtual process $d \rightleftarrows np$. The deuteron is stable in the usual sense because $M_d < M_p + M_n$. For states below threshold, with energies $|\omega_i| < M_i$, a virtual decay can take place if all the particles have positive imaginary momenta $(+i\kappa)$ in the z direction, say. The four-momentum vector of a particle with imaginary three-momentum is Euclidean: $M^2 = \omega^2 + \kappa^2$. The energy momentum conservation equation can be represented geometrically by a triangle in the ω-κ plane as in Fig. 2-6. For the virtual decay to occur all the energies ω and pseudomomenta κ must be positive, which means the triangle will close if $M_d^2 > M_p^2 + M_n^2$. Hence an anomalous singularity will occur for the deuteron because the deuteron mass satisfies

$$(M_p + M_n)^2 > M_d^2 > M_p^2 + M_n^2. \tag{2.28}$$

For the deuteron this anomalous threshold lies very near the physical region, at

$$\begin{aligned} t_0 &= 4M_p^2 - \frac{(M_d^2 - M_p^2 - M_n^2)^2}{M_d^2} \\ &\approx 0.03(\mathrm{GeV}/c)^2. \end{aligned} \tag{2.29}$$

In most phenomenological studies the full complications of kinematics (in particular, of the anomalous threshold) have been ignored. As an example we cite the analysis of coherent $K^*(890)$ production $Kd \to K^*d$ at 4.5 GeV/c of Eisner et al. (1968), in which the

deuteron is treated as a structureless spin-1 object. Typically statistics have been so low that more sophisticated analysis would be unwarranted. For example, see Buchner et al. (1969) for coherent K^* production at 3 GeV/c. Alberi and Bertocchi (1969a) estimated the contribution of inelastic intermediate meson states in $\pi d \to \pi d$. Again the subtleties of kinematics were ignored as the Regge pole parametrization was used to give the Phragmén-Lindelöff theorem connection between asymptotic energy dependence and the phase of an amplitude. Given the success of theories for $\pi d \to \pi d$ which take proper account of spin [cf. Sec. 1.3], the corrections due to inelastic intermediate states are likely to be small.

An exception to the general rule is the paper by Barger and Michael (1969) in which the full gore of Lee's kinematics is applied to $pp \to \pi^+ d$, despite the relative absence of data. An exercise which seems useful for the future is the construction of kinematically correct Regge pole amplitudes for the impulse and eclipse terms for coherent production off deuterons.

2.3. Some Experimental Tests

As we have indicated in the first part of this review, Glauber theory has been tested and refined extensively for elastic hadron-deuteron scattering. Such detailed comparison of theory with experiment has not yet been made in the inelastic cases, and we therefore wish to close by making some simple remarks about inelastic scattering. Little is known about the catastrophic case in which the deuteron is broken up and one of the constituent nucleons is transformed into a nucleon

resonance or a hyperon.* A purely experimental investigation which we can recommend is the comparison of N^* production cross sections off deuterons with the corresponding cross sections off protons. For example, examination of

$$K^+d \rightarrow K^0\Delta^{++} n_s \quad \text{vs} \quad K^+p \rightarrow K^0\Delta^{++}$$

will reveal whether the neutron is truly a spectator or not. If not, the rescatterings $\Delta^{++}n \rightarrow \Delta^{++}n$, etc. may be important effects. This kind of information is needed for one critically to assess the evidence for $I = 2$ exchange reported in a comparison of $\gamma p \rightarrow \pi^{\pm}\Delta$ with $\gamma d \rightarrow \pi^{\pm}\Delta N_s$. [See the discussion by Diebold(1969).]

Backward hadron-deuteron scattering is a case in which the Glauber approximation restricted to small angles would presumably break down. The most straightforward reaction is $pd \rightarrow dp$, for which Bertocchi and Capella (1967) proposed a double scattering mechanism with nucleon exchange to obtain satisfactory agreement with the data of Coleman et al. (1966). No single (known) particle exchange is allowed in $\pi d \rightarrow d\pi$, so any explanation of this reaction will suffer all the ambiguities of exotic box graphs for hadron-hadron scattering.

Coherent excitation of the projectile is a more tractable problem theoretically, and several experiments seem feasible. Of these we mention in particular

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* We thank Dr. G. C. Fox for reminding us of this state of affairs.

(i) $\pi d \to A_1 d$,

(ii) $pd \to N^*(1688)d$,

(iii) $Kd \to Qd$.

All of the final states may be obtained by vacuum exchange from the initial states. Using the multiple scattering formalism, we can formulate the problem to show explicitly what is to be learned from this class of experiments.

For a general coherent production

$$X + d \to X^* + d$$

we generalize the multiple scattering expansion (1.56) in an obvious way to write

$$T = T_p + T_n + E_{X^*n} G_{X^*} T_p + E_{X^*p} G_{X^*} T_n + T_n G_X E_{Xp} + T_p G_X E_{Xn} + \cdots, \tag{2.30}$$

where E_{ij} describes the elastic scattering of particles i and j and T_k is the amplitude corresponding to the process $Xk \to X^*k$. For applications we wish to assume in the spirit of Harrington (1969b) that the infinite series implied by (2.30) can be replaced by the on-shell contributions to the terms we have written explicitly. Then for the reactions (i)-(iii) above everything is known (or otherwise measurable) except the X^*-nucleon elastic scattering amplitude. Thus diffractive excitation of hadron resonances off deuterons becomes a means for studying unstable hadron-nucleon scattering.

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FIGURE CAPTIONS

Fig. 1-1. The two types of diagrams which contribute to elastic hadron-deuteron scattering in the high-energy diffraction theory. [See Eq. (1.30).] (a) Single scattering diagram; (b) double scattering diagram. Another single scattering diagram with proton and neutron interchanged also contributes to the scattering.

Fig. 1-2. The various processes which contribute to charge-exchange scattering by the deuteron in the case of a positively charged incident hadron of isotopic spin 1/2.

Fig. 1-3. The double charge-exchange process.

Fig. 1-4. The total and absorption cross sections for antiproton-deuteron scattering. From Franco and Glauber (1966).

Fig. 1-5. The contributions to proton-deuteron elastic scattering from the single and double scattering terms in the region 15 to 20 GeV/c. From Glauber (1969).

Fig. 1-6. The elastic proton-deuteron data of Bennett et al. (1967), showing the absence of a dip in the "intermediate" region of momentum transfers.

Fig. 1-7. Comparison of the theoretical calculations of Franco and Glauber (1968) with the proton-deuteron elastic scattering experiments (a) at 1 GeV by Bennett et al. (1967); (b) at 2 GeV by Coleman et al. (1966).

Fig. 1-8. Comparison of the theoretical calculations by Alberi and Bertocchi (1969b) with the π^--deuteron elastic scattering

data at 895 MeV/c of Bradamente et al. (1968). The dashed curve corresponds to a pure S-wave deuteron wave function. The solid curve takes the D wave into account.

Fig. 1-9. Comparison of the theoretical calculations of Michael and Wilkin (1969)--solid curves--with the data of Fellinger et al. (1969)--upper three curves--and Bradamente et al. (1970).

Fig. 1-10. Diagram corresponding to the contribution of an inelastic intermediate state for elastic scattering.

Fig. 2-1. Reggeon exchange diagram for X-Y elastic scattering, which is governed by Pomeranchuk (P) exchange.

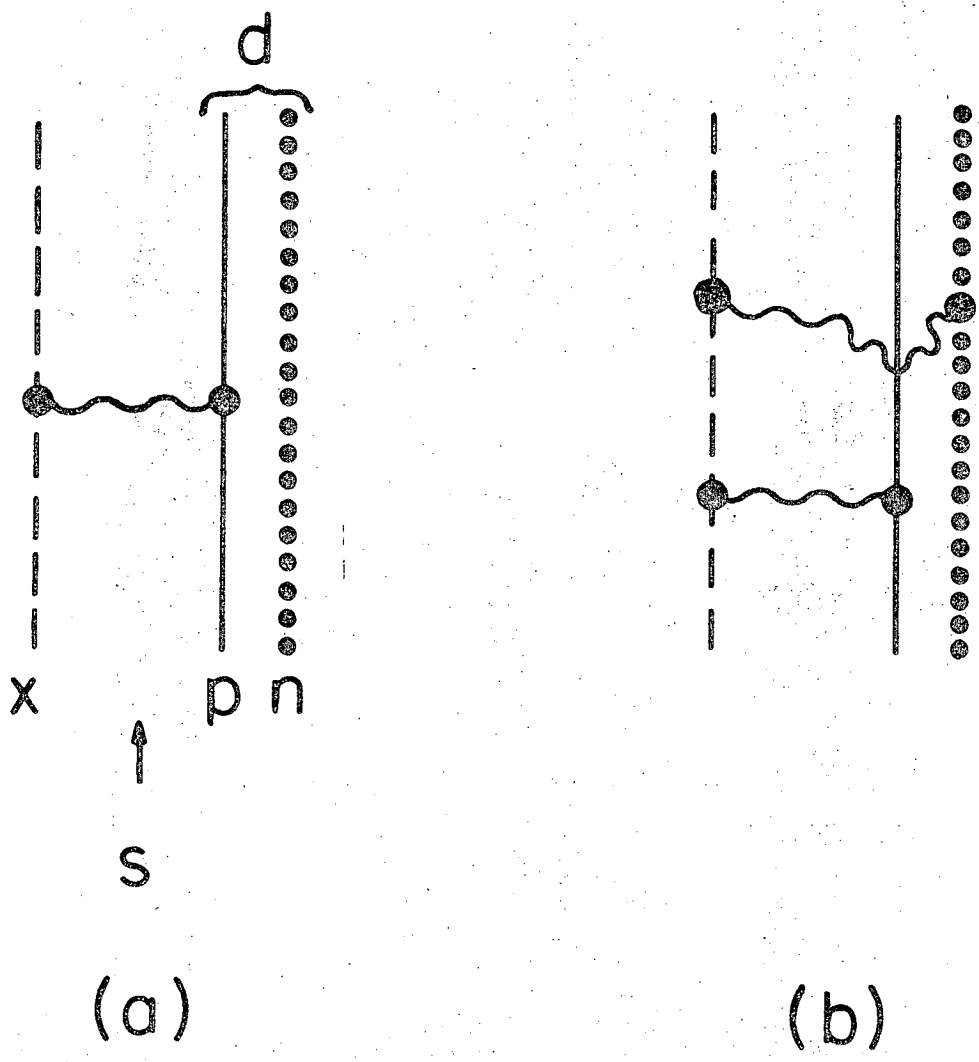
Fig. 2-2. Graphical representation of the Glauber series for hadron (dashed line) - deuteron scattering: (a) and (b) impulse terms; (c) eclipse term. The wavy lines are Regge poles, the solid line the proton and the dotted line the neutron.

Fig. 2-3. General Feynman graph for two-Reggeon exchange in (quasi) two-body scattering. The blobs may have complicated structure.

Fig. 2-4. (a) The simplest Feynman graph which has a Regge cut; (b) redrawn for hadron-deuteron scattering.

Fig. 2-5. Triple scattering Feynman graphs which appear in the Born series for the Glauber eclipse term.

Fig. 2-6. The virtual dissociation $d \leftrightarrows np$ for imaginary momenta of the three particles. The length of a vector is proportional to the mass of the corresponding particle.



XBL706-3226

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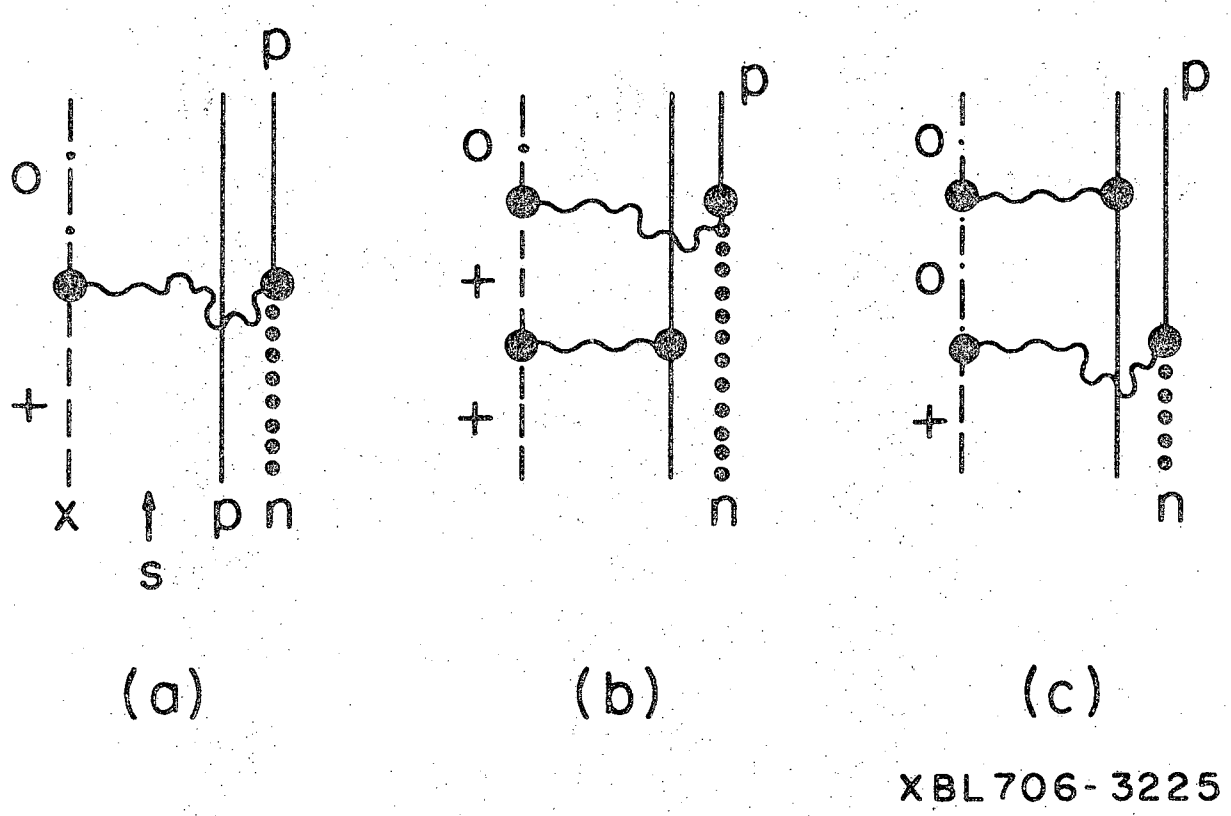

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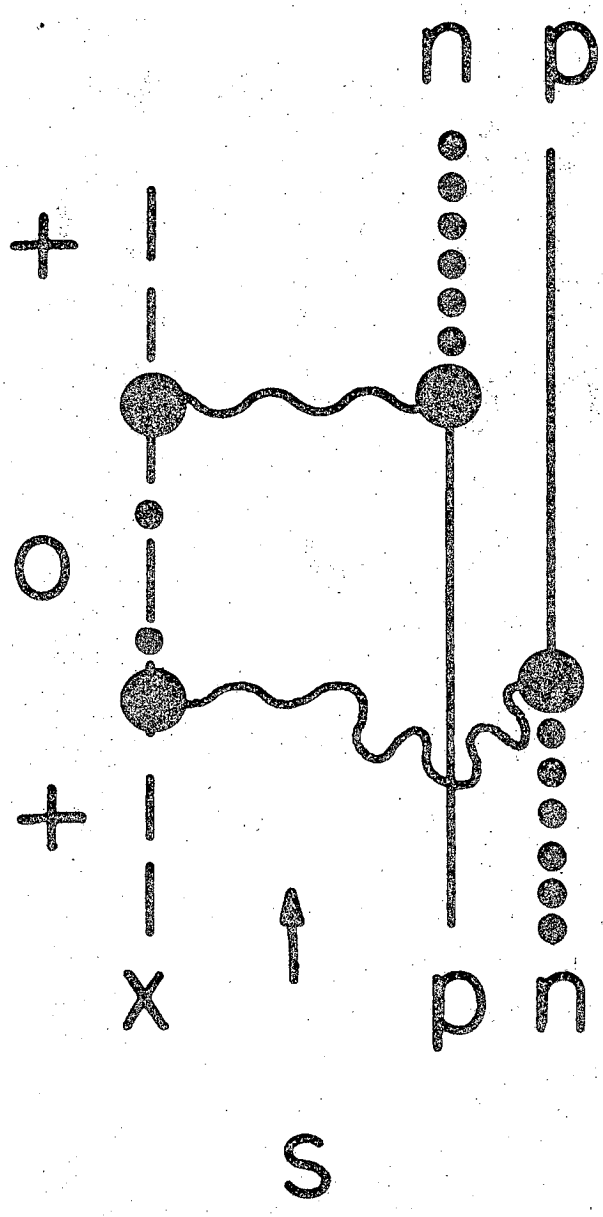

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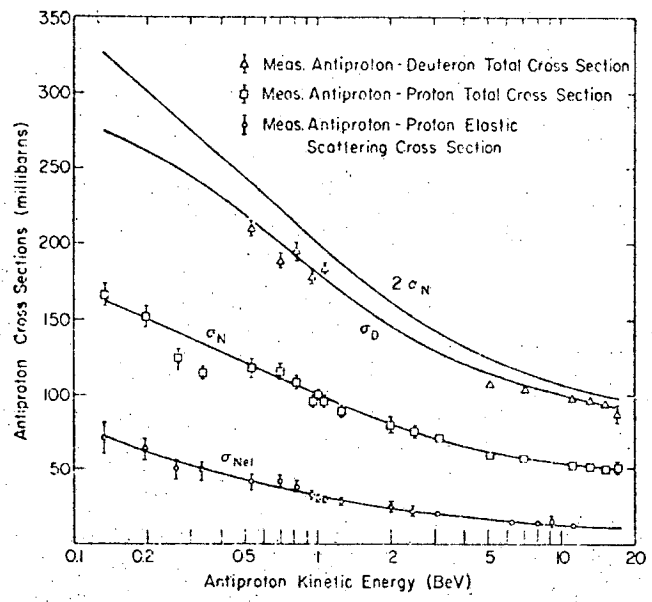

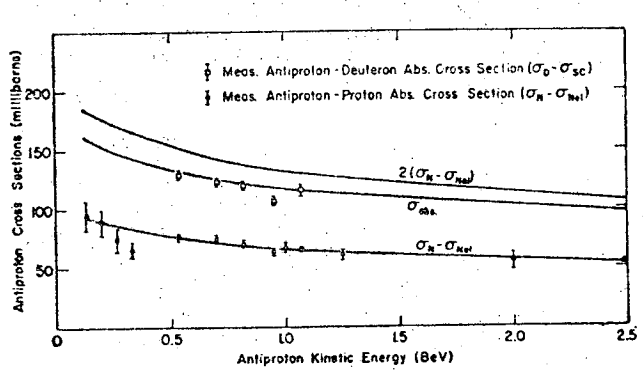

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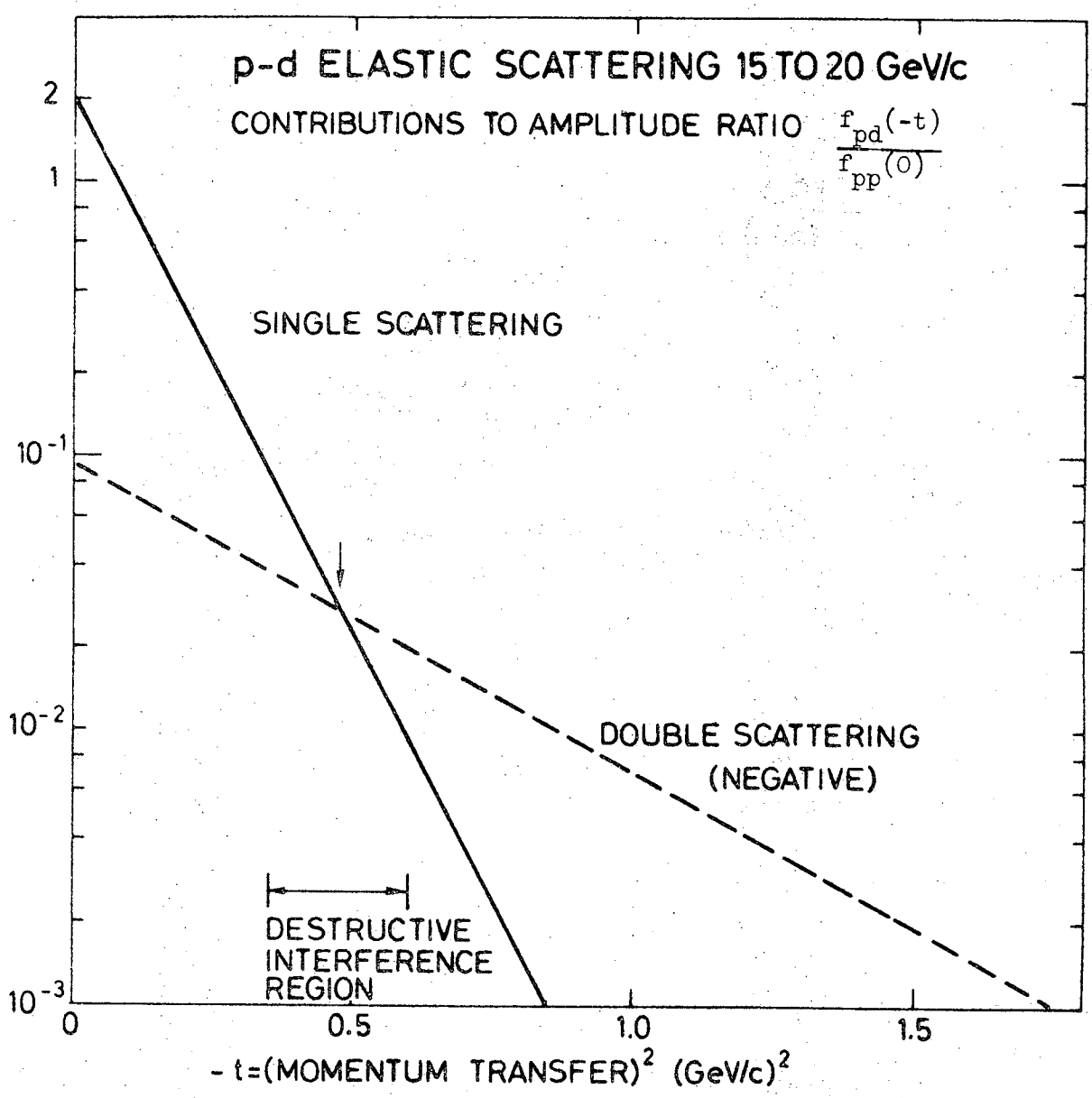

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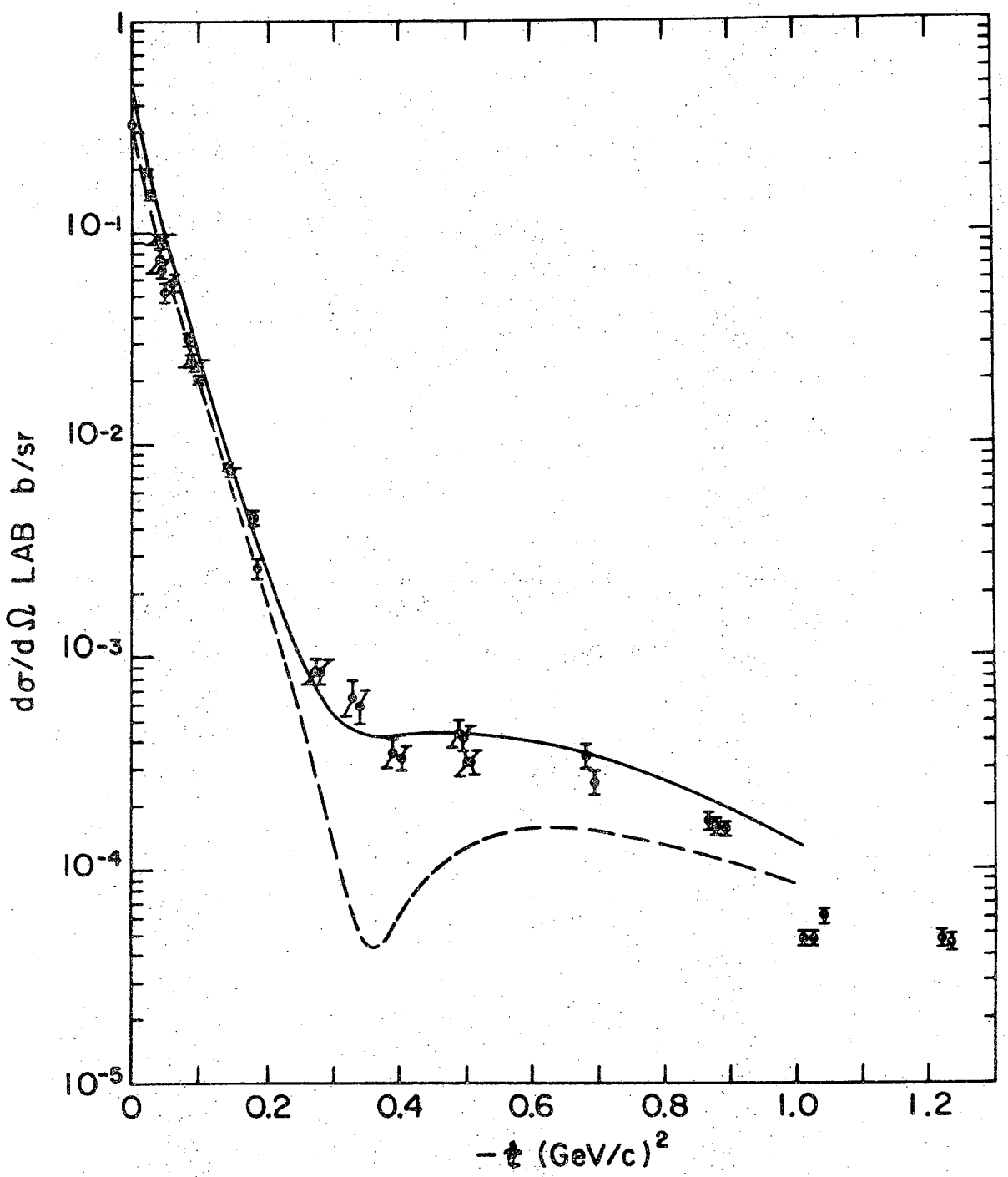

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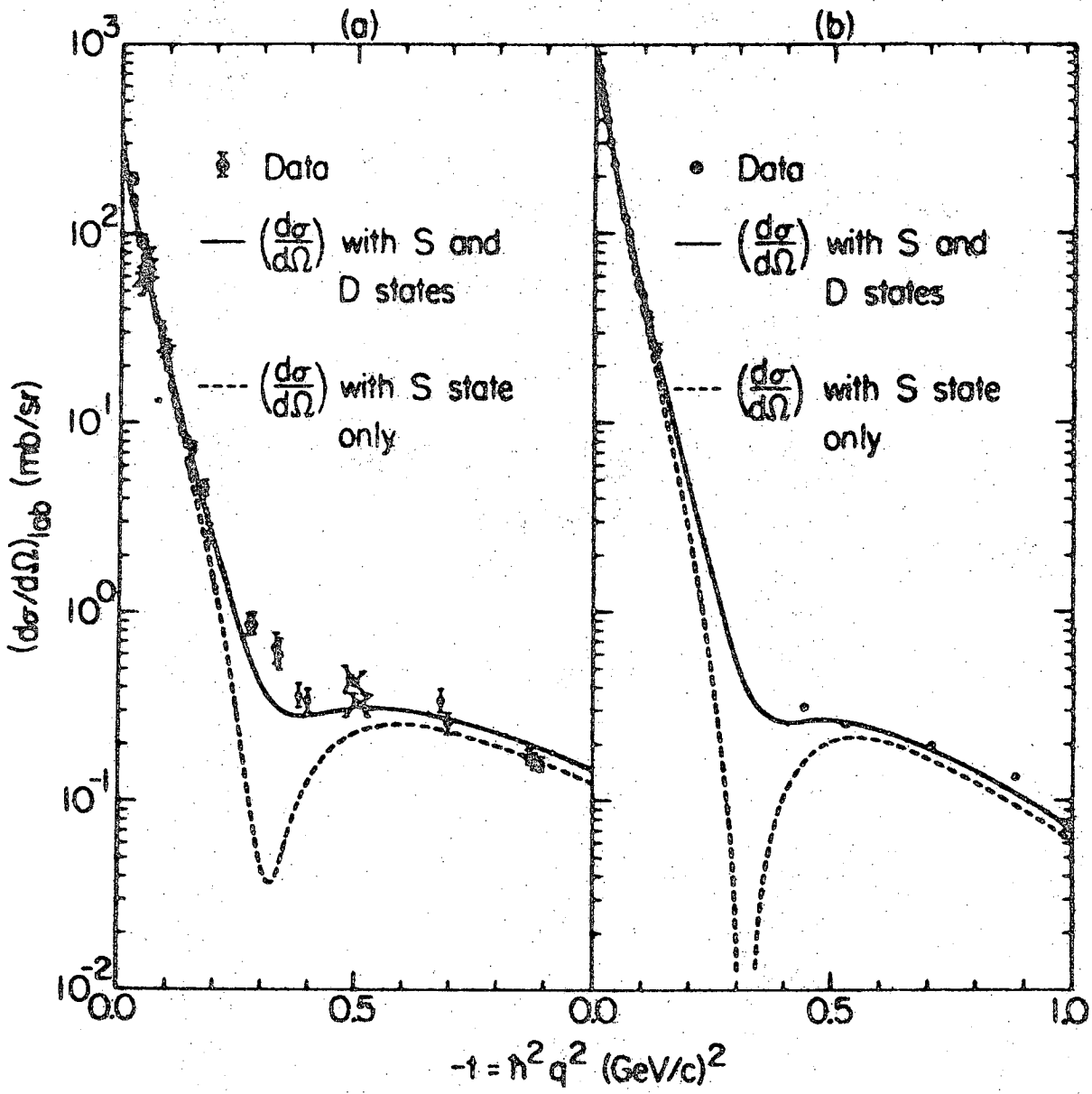

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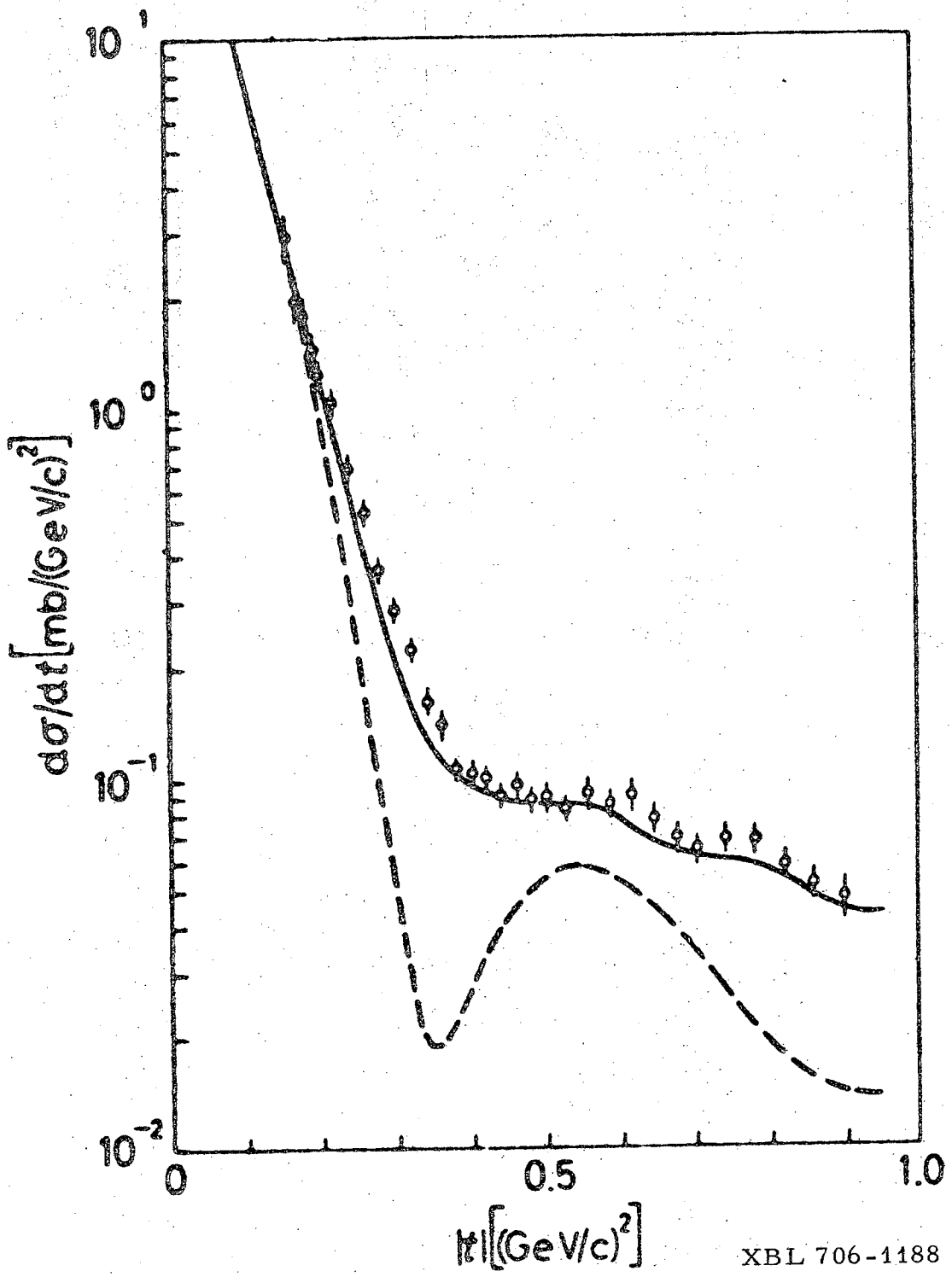

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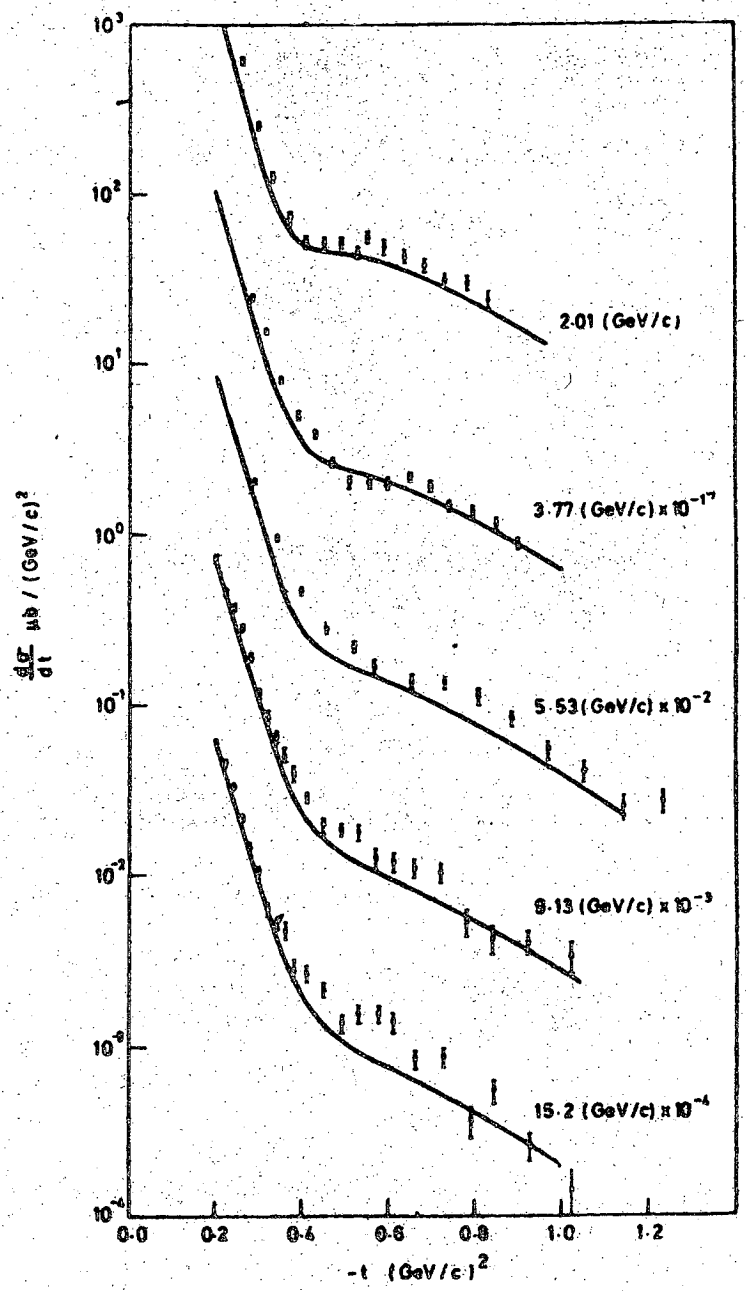

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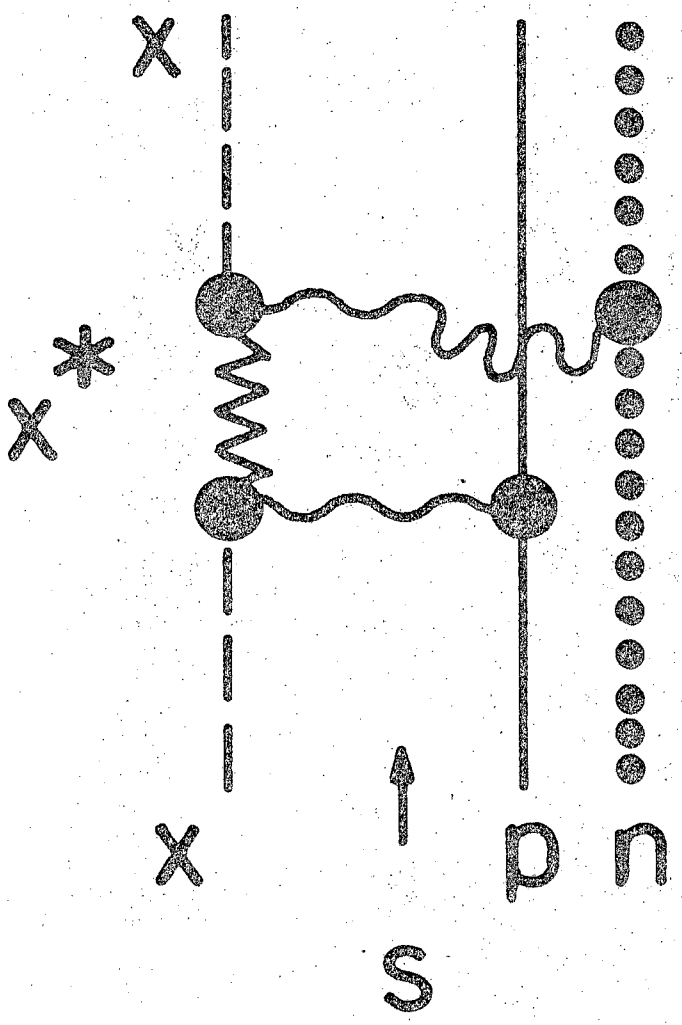

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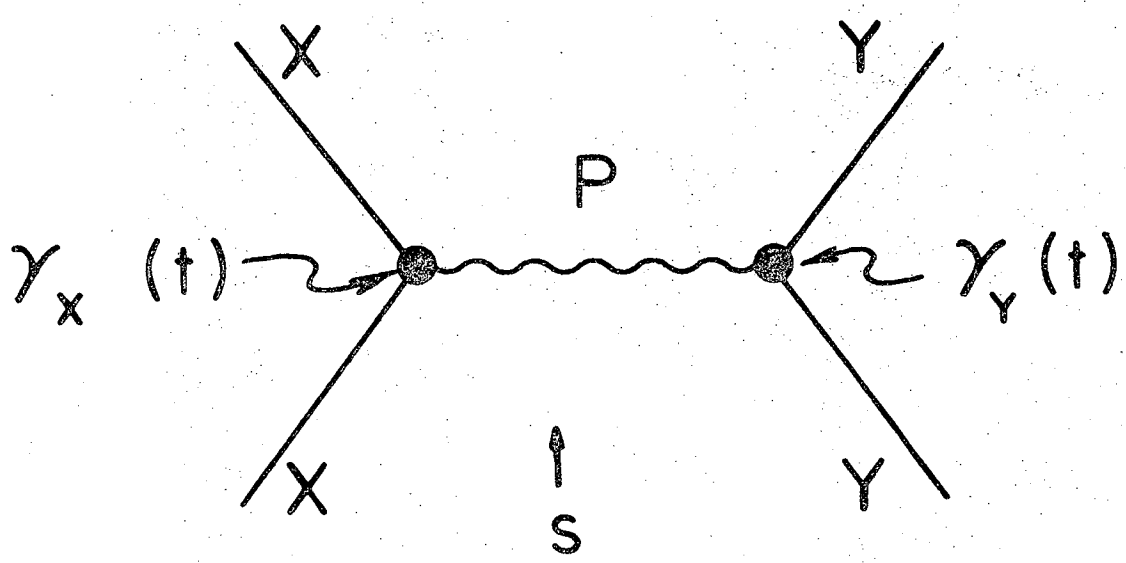

XBL706-3222

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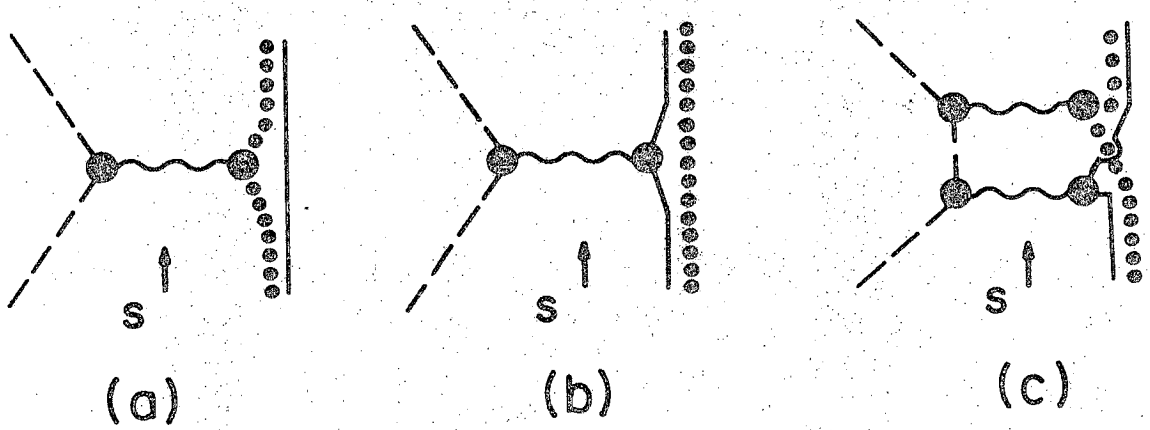

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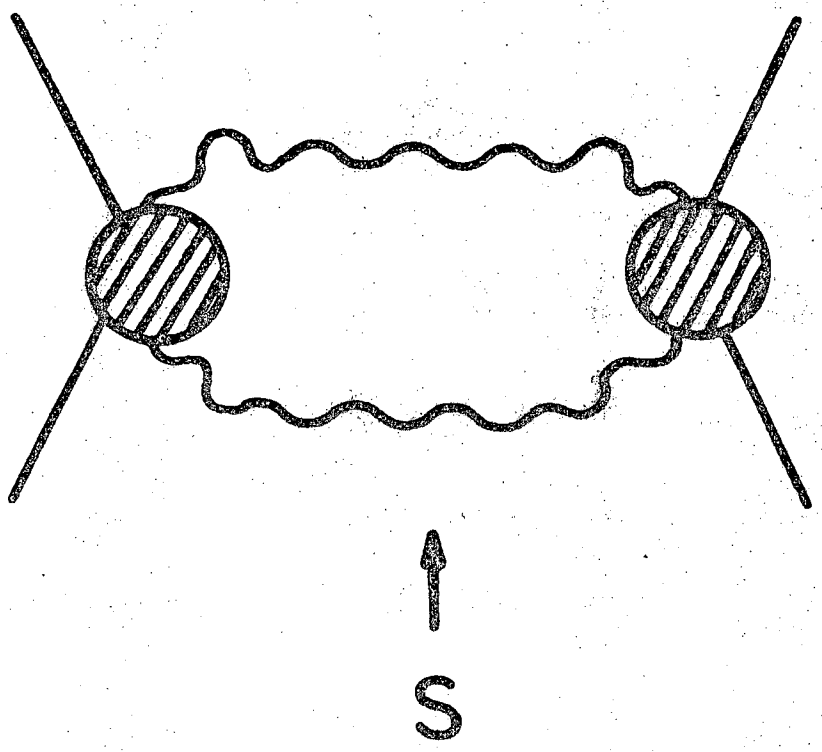
Fig. 2-2



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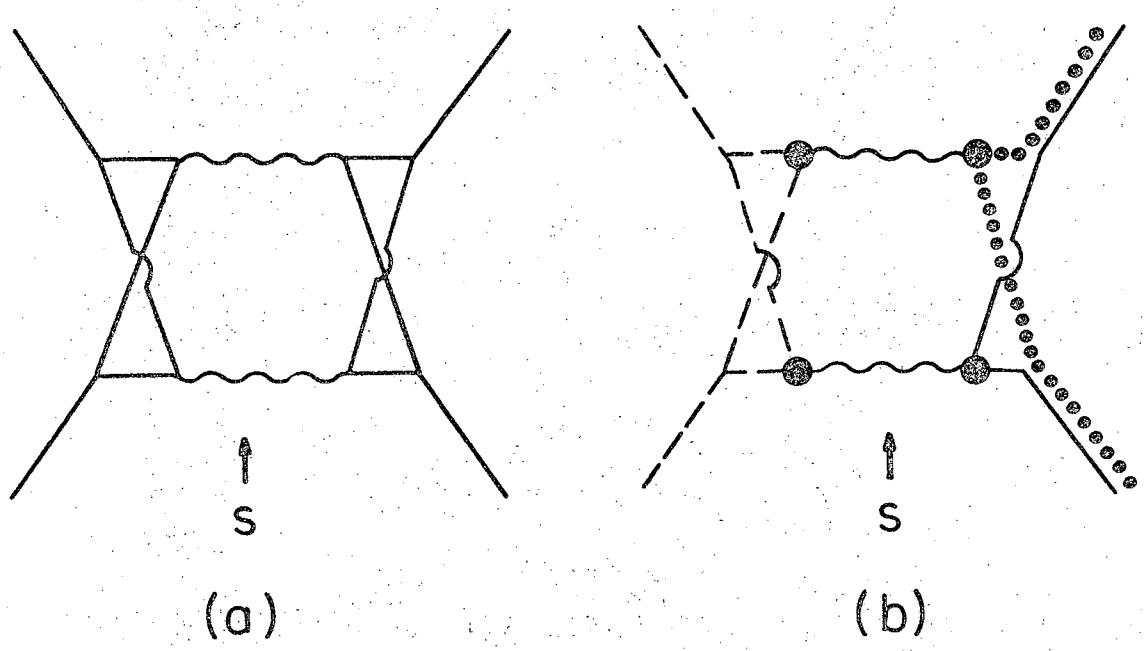

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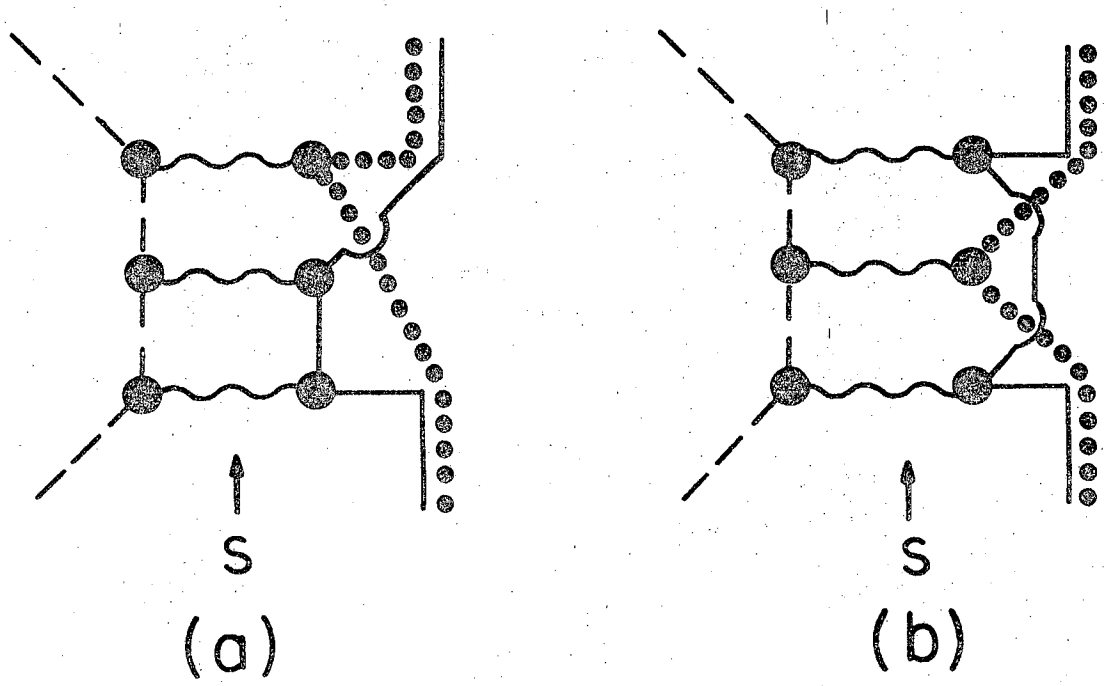

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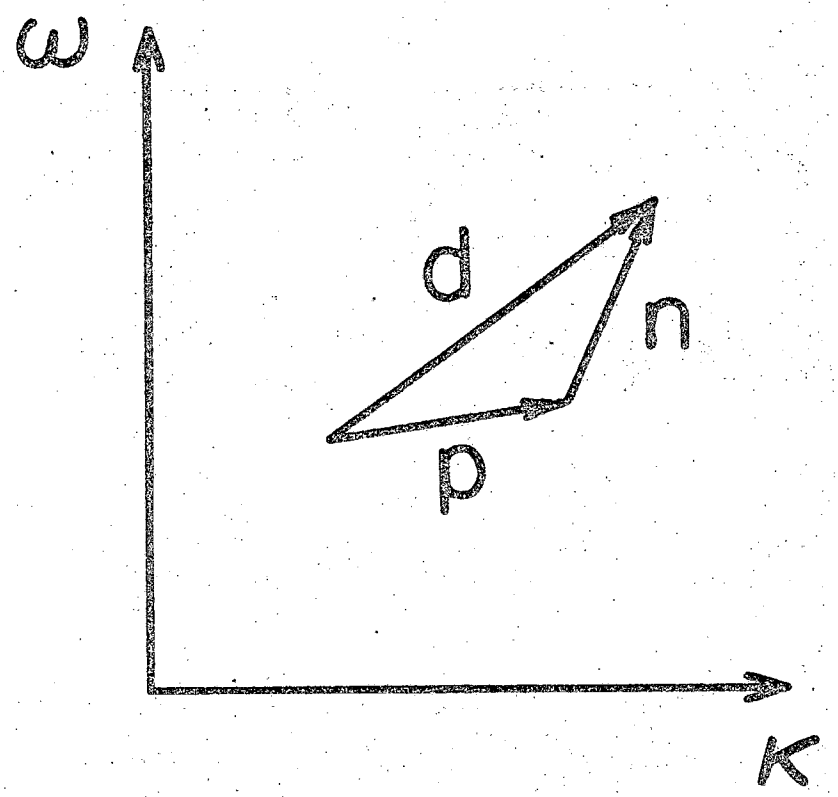
Fig. 2-5



XBL706-3218

Fig. 2-6

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UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720