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## **Simplified Methods for Combining Natural and Mechanical Ventilation**

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## **ABSTRACT**

In determining ventilation rates, it is often necessary to combine naturally-driven ventilation, such as infiltration, with mechanical systems. Modern calculation methods are sufficiently powerful that this can be done from first principles with time varying flows, but for some purposes simplified methods of combining the mechanical and natural ventilation are required—we call this “superposition”. An example of superposition would be ventilation standards that may pre-calculate some quantities within the body of the standard. When there are balanced mechanical systems, the solution is simple additivity, because a balanced system does not impact the internal pressure of the space. Unbalanced systems, however, change internal pressures and therefore can impact natural ventilation in such a way as to make it sub-additive. Several sub-additive superposition models are found in the literature. This paper presents the results of millions of hours of simulations of the physically correct solution, which span a broad range of climates, air leakage and structural conditions. This wide range of data allows for the comparison of three superposition models from the literature and eight new ones. The results show that by using the appropriate model(s) superposition errors can be reduced significantly, from the 20% over-prediction of simple linear addition to 1% or less.

## **KEYWORDS**

Unbalanced ventilation, infiltration, REGCAP simulation, empirical models, superposition

## 1. INTRODUCTION

Most homes are ventilated by the form of natural ventilation known as “infiltration”, which is defined in ASHRAE Standard 62.2 (ASHRAE 2013) as the “uncontrolled inward leakage of air through cracks and interstices in any building element and around windows and doors of a building”. In order to decrease energy consumption, house envelopes are getting tighter. Combined with potential increases in pollutant sources in indoor living environments, this raises concerns for indoor air quality (IAQ). People spend an average of 90% of their time inside. With the increased concern over IAQ, more houses are using a mechanical ventilation system to maintain a good air quality.

If we wish to optimize the dual concerns of acceptable IAQ and minimum energy consumptions it is important to understand the total ventilation rate and that means it is important to understand how to combine natural ventilation such as infiltration with mechanical ventilation—i.e. fans. Detailed simulations models provide this capability, but often it is desirable to have simplified approaches that allow pre-calculation of certain quantities and then an after-the-fact superposition.

Infiltration is caused by two driving forces, namely the wind and stack effects. The wind raises the pressure on the windward side of the building, and lowers the pressure on the other sides in proportion to the square of wind speed. The stack effect refers to density differences between indoor and outdoor air resulting from differences in temperature. In winter, the heated air inside the building is less dense than the cold air outside resulting in pressure differences across the envelope with higher inside pressure at the top of the building and lower inside pressure at the bottom of the building. The reverse happens in summer when the outside temperature is greater than the inside.

If a balanced ventilation system is installed, the impact on infiltration will not be significant, because the balanced system does not change the pressures across the building leaks. As a result, the total ventilation rate ( $Q_t$ ) is simply the addition of the fan flow ( $Q_f$ ) and the natural infiltration ( $Q_{inf}$ ).

Unbalanced mechanical ventilation systems modify the indoor pressure of the building, which interacts with the wind and stack induced flows, making the combination of the flows sub-additive. Exhaust fans depressurize the building, which increases the airflow in through the building envelope. The greater the fan flow, the higher proportion of the building envelope experiences inflow. The opposite effect occurs with supply-only systems.

In order to avoid both excessive energy consumption and poor IAQ, it is necessary to predict the total flow rate resulting from the combined natural and mechanical ventilation. This can be done using mass balance physical and mathematical models to find the internal pressure that balances the incoming and outgoing mass flows. This approach is powerful, but requires many computational inputs and can be too time consuming for some purposes such as ventilation standards or simplified parametric modeling. An alternative is to use a simple empirical model for estimating the total ventilation rate  $Q_t$  from  $Q_f$  and  $Q_{inf}$ . These models are generically called “superposition” models. A few models were suggested and tested a few decades ago, but the results were sometimes contradictory. There is to date no clear “winner”, but the model that is in the ASHRAE Handbook of Fundamentals (i.e. simple quadrature) can be considered the closest thing to a consensus. Another application is that used in ASHRAE Standard 62.2, in which the total required flow is known ( $Q_t$ ) together with an estimate of natural infiltration ( $Q_{inf}$ ) and we wish to determine the appropriate fan flow ( $Q_f$ ) to reach the total.

In this study, we used the REGCAP model to simulate millions of hours of the physically correct solution to determine the infiltration alone and the combined ventilation with mechanical systems (i.e., full mass balance and pressure balancing), with a broad range of climate, air leakage and structural conditions. Then we compared this data with three superposition models from the literature as well as 8 new models, including empirical ones based on the simulation results. The objective was to determine the uncertainty of existing models and to develop improved models that retain the ideal of simplicity.

## 2. BACKGROUND

### Previous work on superposition

In the eighties and early nineties a number of models for empirically combining the natural infiltration flow and unbalanced mechanical ventilation were suggested. A summary is presented in Table 1.

Most of the models assume a linear addition of pressure differences:

$$\Delta P_t = \Delta P_{inf} + \Delta P_f \quad (1)$$

This is correct locally at the leakage scale, but is only an approximation at the building scale, since at some locations positive and negative pressures will cancel each other out.

Moreover the flow rate  $Q$  induced by a pressure difference  $\Delta P$  is given by:

$$Q = C\Delta P^n \quad (2)$$

Where  $C$  ( $m^3/(hPa^n)$ ) is the leakage coefficient characterizing the air permeability, and  $n$  (-) is the flow exponent, ranging from 0.5 to 1, which gives information on the flow regime (i.e., laminar versus turbulent). Values close to 0.5 correspond to turbulent flows obtained with large leaks such as orifices, whereas values close to 1 indicate laminar flow conditions. For a building envelopes,  $n$  is normally found to be in the vicinity of 0.65.

By adapting equation (2) to the total ( $Q_t$ ), infiltration ( $Q_{inf}$ ) and fan flows ( $Q_f$ ), and by considering  $C$  as a constant, (2) becomes:

$$Q_t^{\frac{1}{n}} = Q_{inf}^{\frac{1}{n}} + Q_f^{\frac{1}{n}} \quad (3)$$

For  $n=1$ , this is the additivity model that will always over-predict the real total ventilation rate:

$$Q_t = Q_{inf} + Q_f \quad (4)$$

For  $n=0.5$  (orifice flow), this is the quadrature model which is the current model in ASHRAE Handbook of Fundamentals:

$$Q_t = \sqrt{Q_{inf}^2 + Q_f^2} \quad (5)$$

If  $n$  is left as the real flow exponent, which differs slightly from one building to another:

$$Q_t = \left( Q_{inf}^{\frac{1}{n}} + Q_f^{\frac{1}{n}} \right)^n \quad (6)$$

There are many other methods for empirically combining the natural infiltration flow and mechanical ventilation (see the review by Li (1990)). However, many of these are optimized for limited situations, such as the Palmiter and Bond (1991) method, referred to here as the half-fan model, which was developed for stack-only natural infiltration.

Li tested ten models by comparing them with a flow model over a range of wind speeds (0 to 8 m/s) and temperature differences (-20 to 20°C) with open and closed exterior doors and two different exhaust fan speeds. His conclusion was that the quadrature combination of natural and mechanical ventilation worked best. This result is in agreement with the earlier work of Modera and Peterson (1985), who also used a mass balance ventilation model.

Field tests with tracer gas measurements by Kiel and Wilson (1987) found that for strong exhaust mechanical ventilation (four times the natural rate), simple linear addition was the most acceptable method, but that from a theoretical point of view, a half-pressure addition and half-linear addition model had more appeal with similar results to the linear addition. (See Table 1 for model definitions.) Continuing this work, Wilson and Walker (1990) looked at a reduced fan flow rate that was approximately equal to the natural rate. The result was the same as Kiel and Wilson, where linear and half-linear/half-pressure addition were the closest to the measured and modeled combined rates. The above two studies looked at exhaust fans only, but over a wide range of natural infiltration driven by both wind and stack effects. Unlike Li, these studies showed large underpredictions using quadrature. This could be due to different building envelope leakage, weather conditions, leakage distributions and strengths of mechanical ventilation, but it mainly underlines the necessity of additional study.

## REGCAP Software

REGCAP is a two zone ventilation model combined with a heat transfer model and a simple moisture transfer model. The two zones are the house and the attic above it. The ventilation rate is found by

**Table 1: Summary of the suggested models and the results of the simulation/experimental comparison studies carried out on them**

Name/Ref	Model	Range	Comparison		
			Ref.	Sim/Exp	Results
Additivity	$Q_t = Q_f + Q_{inf}$	All	Kiel & Wilson	Exp.	best agreement
			Wilson & Walker	Exp.	overpredicts $Q_t$ by 7%
			Li	Sim.	average error: 33%; max. error: 64%
Quadrature	$Q_t = \sqrt{Q_f^2 + Q_{inf}^2}$	All	Modera & Peterson	Sim.	good agreement: error on $Q_t < 10\%$
			Kiel & Wilson	Exp.	underpredicts $Q_t$ by 15-30%
			Wilson & Walker	Exp.	Underpredicts $Q_t$ by 20%
			Li	Sim.	good agreement: average error: 5%; max. error: 17%
			Palmiter & Bond	Exp.	underpredicts for $Q_{inf} < Q_f$ ; overpredicts for $Q_{inf} > Q_f$
Levins (1982)	$Q_t = Q_{inf} + Q_f \cdot \exp\left(-\frac{Q_{inf}}{Q_f}\right)$	All	Kiel & Wilson	Exp.	underpredicts $Q_t$ by 15-30%
			Li	Sim.	good agreement: average error: 5%; max. error: 20%
Power Law	$Q_t = \left(Q_f^{\frac{1}{n}} + Q_{inf}^{\frac{1}{n}}\right)^n$	All	Modera & Peterson	Sim.	bigger errors on $Q_t$ than the quadrature model
			Kiel & Wilson	Exp.	underpredicts $Q_t$ by 10-25%
			Li	Sim.	average error: 11%; max. error: 30%
Shaw (1985)	$Q_t = \begin{cases} Q_f & \text{for } h_0^1 > H \\ F \left( Q_{w-f}^{\frac{1}{n}} + Q_w^{\frac{1}{n}} \right)^n & \text{for } h_0 < H \end{cases}$		Shaw	Exp.	in general within 25% of the measured values
			Kiel & Wilson	Exp.	underpredicts $Q_t$ by 15-30%
Kiel	$Q_t = \sqrt{Q_f^2 + (2Q_{inf})^2}$	$Q_f \gg Q_{inf}$	Kiel & Wilson	Exp.	very spread data; overpredicts $Q_t$ when $Q_f < 0.7Q_t$ ; otherwise mostly underpredicts $Q_t$
			Li	Sim.	average error: 56%; max. error: 100%
Li	$Q_t = \left(Q_f^{\frac{1}{n}} + (2Q_{inf})^{\frac{1}{n}}\right)^n$	$Q_f \gg Q_{inf}$	Li	Sim.	average error: 98%; max. error: 160%
Kiel & Wilson	$Q_t = \sqrt{\left(\frac{Q_f}{2}\right)^2 + Q_{inf}^2} + \frac{Q_f}{2}$	All (Exhaust fan)	Kiel & Wilson	Exp.	underpredicts $Q_t$ by 10-30%
			Li	Sim.	average error: 12%; max. error: 35%
			Palmiter & Bond	Exp.	overpredicts the fan efficiency
Wilson & Walker	$Q_t = \left(\left(\frac{Q_f}{2}\right)^{\frac{1}{n}} + Q_{inf}^{\frac{1}{n}}\right)^n + \frac{Q_f}{2}$	All (Exhaust fan)	Wilson & Walker	Exp.	underpredicts $Q_t$ by 7%
			Li	Sim.	average error: 18%; max. error: 42%
Li	$Q_t = \frac{1}{2}\sqrt{Q_f^2 + (2Q_{inf})^2}$	$Q_f < Q_{inf}$	Li	Sim.	average error: 22%; max. error: 50%
Li	$Q_t = \frac{1}{2}\left(Q_f^{\frac{1}{n}} + (2Q_{inf})^{\frac{1}{n}}\right)^n$	$Q_f < Q_{inf}$	Li	Sim.	average error: 21%; max. error: 50%
Half-fan - Palmiter & Bond	$Q_t = \begin{cases} \frac{Q_f}{2} + Q_{inf} & \text{for } Q_f < 2Q_{inf} \\ Q_f & \text{for } Q_f \geq 2Q_{inf} \end{cases}$		Palmiter & Bond	Exp.	good agreement

<sup>1</sup> Height of neutral level compared to ceiling

determining for each zone the internal pressure required to balance the incoming and outgoing mass flows resulting from the natural and mechanical ventilation driving forces.

The model uses an envelope airtightness measurement and a description of the leakage distribution. The leakage for the home is split between walls, floor, ceiling and open flues/chimneys. In this study, the leakage distribution was varied with the number of storeys and the type of foundation. Each leak is defined by its flow coefficient, pressure exponent, height above grade, wind shelter and wind pressure coefficient taken from wind tunnel tests. An iterative numerical method is used to solve the non-linear mass balance equations. The attic temperature is not regulated and will therefore both be affected by the ventilation rate and affect the infiltration flow due to the stack effect. In addition, REGCAP includes models for the HVAC equipment in the home and operates on one-minute time steps. The ventilation and heat transfer models are coupled and the combined solution is also found iteratively. A more detailed discussion of REGCAP, including validation compared to measured field data, can be found in Walker et al. (2005).

## Applications

Each simplified model can either be used for forward or inverse calculations. The forward model predicts the total ventilation airflow ( $Q_t$ ) as a function of the natural infiltration ( $Q_{inf}$ ) and the fan flow ( $Q_f$ ). The equivalent inverse model gives  $Q_f$  as a function of  $Q_t$  and  $Q_{inf}$ . They can be applied to hourly or annual calculations, which results in four different cases:

- Hourly, Forward Case: for the hourly air change rate prediction; useful for estimating energy loads and needed for relative exposure calculations.
- Annual, Forward Case: predicting the annual effective ventilation given the effective infiltration and a fixed (or effective) fan flow; for indoor air quality (IAQ) purposes.
- Hourly, Inverse Case: when one wants to vary the fan size each hour to compensate for varying hourly infiltration in order to keep the total ventilation constant.
- Annual, Inverse Case: for finding the fixed fan size that will combine with effective infiltration to produce a desired total ventilation; useful for building codes/standards applications such as ASHRAE Standard 62.2.

### 3. APPROACH

#### REGCAP simulations

We used REGCAP to create simulation data based on a wide range of weather and housing conditions. The range of inputs is presented in Table 1 and results in 720 combinations. For simplicity a variety of assumptions were made. For example, no (other) exhaust devices (such as kitchen or bathroom exhausts) were used and the duct leakage was set to zero, since these kinds of factors would obscure the underlying impact. The number of stories changes, but the floor area is constant at 1900 ft<sup>2</sup> (176 m<sup>2</sup>). We calculated the flow through the exhaust or supply fan ( $Q_f$ ) according to ASHRAE Standard 62.2, including the infiltration credit in the standard. For each set of inputs, we ran the model for a full year using TMY3 data for the climates shown in Table 2. REGCAP used minute-by-minute time steps and we calculated hourly averages the REGCAP output. This resulted in more than 6.3 million of points of comparison for the superposition models.

Table 2: Range of inputs for the REGCAP simulations

Parameters	Values
Envelope airtightness ( $ACH_{50}$ )	0.6; 3; 5; 7; 10
Mechanical ventilation type	Exhaust ; supply
Number of stories	1; 2; 3
Foundation type	Slab on-grade; crawlspace; basement
Climate zones	Miami; Houston; Memphis; Baltimore; Chicago; Burlington; Duluth; Fairbanks

For each combination of house inputs, three REGCAP simulations were performed. First, we used REGCAP to calculate the infiltration flow through the envelope ( $Q_{inf}$ ) due to the stack and wind effect, with no mechanical ventilation operating. We then repeated the simulations with supply or exhaust fans operating to obtain the total flow ( $Q_t$ ). Then we compared the results for each superposition method for combining  $Q_f$  and  $Q_{inf}$  to  $Q_t$ . The way these airflows are extracted or calculated from the simulation output files is explained in Appendix A.

For the annual calculations, the fan flow is still the same as it is a constant over the year, but  $Q_{inf}$  and  $Q_t$  are effective annual average infiltration rates. The effective rates are the correct ones to used for indoor air quality applications (as shown by Sherman and Wilson (1986) and subsequent work). While

simply averaging ventilation rates can be easier to measure, they do not in fact correspond to anything physically useful. The effective values are almost always lower than the averaged ones. As defined in ASHRAE Standard 62.2, they correspond to “the constant air infiltration rate that would result in the same average indoor pollutant concentration over the annual period as actually occurs under varying conditions”. This annual approach can be particularly useful when one wants to size the fan to the total ventilation required by ventilation standards.

## Simplified models

Superposition is not a new concept. Many researchers have tried in the past to come up with empirical or first principle models. Some of those models work in some situations, but heretofore, none of the models work well enough over a broad range of potential applications. In this section we review eleven models from the literature, models that have been proposed and models developed as a result of this effort. The equations describing the forward and backward forms are presented in Table 3.

- **Models from the literature**

The three first models come from the literature described earlier. The **additivity** model, which is a simple addition of the flows, is in the current ASHRAE 62.2 Standard, and has been experimentally verified by Kiel and Wilson. **Simple quadrature** is the current model in the ASHRAE Handbook of Fundamentals, and has been verified by both Modera & Peterson and Li. The **Half Fan** model was used in earlier editions of ASHRAE Handbook of Fundamental and has been established experimentally by Palmiter and Bond. For each of these models the forward and inverse forms are equivalent. For all three models, verification was for a narrow range of conditions, and the current study aims to investigate their performance over a much wider range of homes and conditions. They are compared with eight new models described below.

- **Advanced quadrature**

The advanced quadrature is similar to the quadrature model but with an additional correction term driven by the coefficient  $\beta$ :

$$Q_t = \sqrt{Q_f^2 + Q_{inf}^2 + \beta Q_f Q_{inf}} \quad (7)$$

If  $\beta=2$  this is the additivity model that will always overpredict  $Q_t$ . If  $\beta=0$  this is the simple quadrature model that tends to underpredict  $Q_t$ . So by taking intermediate values of  $\beta$  we can get better predictions of  $Q_t$ . We have studied two of them:  $\beta=0.3$  and  $\beta=0.6$  which are minimizing respectively the error for the hourly and annual prediction. The way we obtained these two values is explained in Appendix B.

**Table 3: Forward and backward equations of the simplified models compared to the REGCAP results**

Model	Forward	Inverse
Additivity ( $\beta=2$ )	$Q_t = Q_f + Q_{inf}$	$Q_f = Q_t - Q_{inf}$
Simple quadrature ( $\beta=0$ )	$Q_t = \sqrt{Q_f^2 + Q_{inf}^2}$	$Q_f = \sqrt{Q_t^2 - Q_{inf}^2}$
Advanced quadrature	$Q_t = \sqrt{Q_f^2 + Q_{inf}^2 + \beta Q_f Q_{inf}}$	$Q_f = \frac{\sqrt{\beta^2 Q_{inf}^2 + 4(Q_t^2 - Q_{inf}^2)} - \beta Q_{inf}}{2}$
Half-fan	$Q_t = \max\left(Q_f, Q_{inf} + \frac{Q_f}{2}\right)$	$Q_f = \min\left(Q_t, 2(Q_t - Q_{inf})\right)$
Half-smaller	$Q_t = \max\left(Q_f + \frac{Q_{inf}}{2}, Q_{inf} + \frac{Q_f}{2}\right)$	$Q_f = \min\left(Q_t - \frac{Q_{inf}}{2}, 2(Q_t - Q_{inf})\right)$
System coefficient ( $1/D=0.85$ )	$Q_t = \frac{1}{D} Q_f + Q_{inf}$	$Q_f = D(Q_t - Q_{inf})$
Simple forward sub-additivity (SFSA)	$Q_t = Q_f + \frac{Q_{inf}^2}{Q_{inf} + Q_f}$	$Q_f = \frac{(Q_t - Q_{inf}) + \sqrt{Q_t^2 + 2Q_t Q_{inf} - 3Q_{inf}^2}}{2}$
Simple inverse sub-additivity (SISA)	$Q_t = \frac{Q_f}{2} + \sqrt{\frac{Q_f^2}{4} + Q_{inf}^2}$	$Q_f = Q_t - \frac{Q_{inf}^2}{Q_t}$
Exponential forward sub-additivity (EFSA)	$Q_t = Q_f + \exp\left(-k_{fw} \frac{Q_f}{Q_{inf}}\right) Q_{inf}$	-
Exponential inverse sub-additivity (EISA)	-	$Q_f = Q_t - \exp\left(-k_{inv} \left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf}$
Modified Levins sub-additivity (MLSA)	$Q_t = \exp\left(-k'_{fw} \frac{Q_f}{Q_{inf}}\right) Q_f + Q_{inf}$	-

**Table 4: Values of the k coefficient for the exponential models**

	Forward		Inverse
	EFSA	MLSA	EISA
<b>Hourly</b>	$k_{fw,h} = \frac{2}{3}$	$k'_{fw,h} = \frac{2}{3}$	$k_{inv,h} = 1$
<b>Annual</b>	$k_{fw,a} = \frac{4}{9}$	$k'_{fw,a} = \frac{1}{2}$	$k_{inv,a} = \frac{2}{3}$

- **Half-smaller**

The half-fan model assumes a stack-driven infiltration since the total ventilation equals to  $Q_f$  for strong mechanical ventilation. In the example of an exhaust fan, it corresponds to a situation where the neutral pressure level rises above the ceiling level. There is no exfiltration through the building envelope, and the infiltration is therefore only compensating for the exhaust fan flow. But the stronger the wind, the less likely this is to happen. As a result, when assuming a certain independence of the infiltration from the fan flow, a logical extension would be the following model:

$$Q_t = \max\left(Q_f + \frac{Q_{inf}}{2}, Q_{inf} + \frac{Q_f}{2}\right) \quad (8)$$

- **System Coefficient**

The ASHRAE 62.2 committee is investigating a new model as a replacement of the additivity one:

$$Q_t = \frac{1}{D} Q_f + Q_{inf} \quad (9)$$

D is the system coefficient, with a value suggested of 1/0.85 for not balanced ventilation systems. More details about this model can be found in Appendix C.

- **Sub-additivity models**

We used the simulation results to approximate a sub-additivity coefficient ( $\Phi$ ) weighting the infiltration contribution to either the total ventilation (forward models) or the fan sizing (inverse models):

$$\begin{cases} Q_t = Q_f + \Phi Q_{inf} \\ Q_f = Q_t - \Phi Q_{inf} \end{cases} \quad (10)$$

$$\Phi = \frac{Q_t - Q_f}{Q_{inf}} \quad (11)$$

We chose this structure because infiltration is often viewed as a “credit” towards the total ventilation. That credit will clearly vary as a function of leakage and either fan size or target ventilation. The sub-additivity coefficient non-dimensionalizes the effect and we can then see the effect in terms of other non-demensionalized values (specifically the fraction of the total ventilation that infiltration provides on its own.) Examining the results in this non-dimensional way allows observation of the physical trends

abstracted from the specifics of any one house. In order to find simple models to approximate this coefficient we examine it as a function of the ratio of known flow rates. We chose the infiltration fraction,  $\alpha$ , for the inverse model and a slightly different version for the forward one since  $Q_t$  is unknown:

$$\begin{cases} \alpha = \frac{Q_{inf}}{Q_t} \\ \alpha_{fw} = \frac{Q_{inf}}{Q_f + Q_{inf}} \end{cases} \quad (12)$$

As shown in Figure 1 the values of  $\Phi$  are following an almost linear trend except for very low infiltration rates for which the total ventilation flow seems to be equal to the fan flow. This exception suggests that at very low infiltration rates, infiltration has no effect. The half-fan model has this behavior qualitatively, but has a functional form overall. If, for simplicity, we ignore that effect (since the errors may not be significant),  $\Phi$  can be approximated by the infiltration fraction. For the forward case this yields the **simple forward sub-additivity (SFSA)** model:

$$Q_t = Q_f + \frac{Q_{inf}^2}{Q_f + Q_{inf}} \quad (13)$$

For the inverse calculations it yields the **simple inverse sub-additivity (SISA)** model:

$$Q_f = Q_t - \frac{Q_{inf}^2}{Q_t} \quad (14)$$

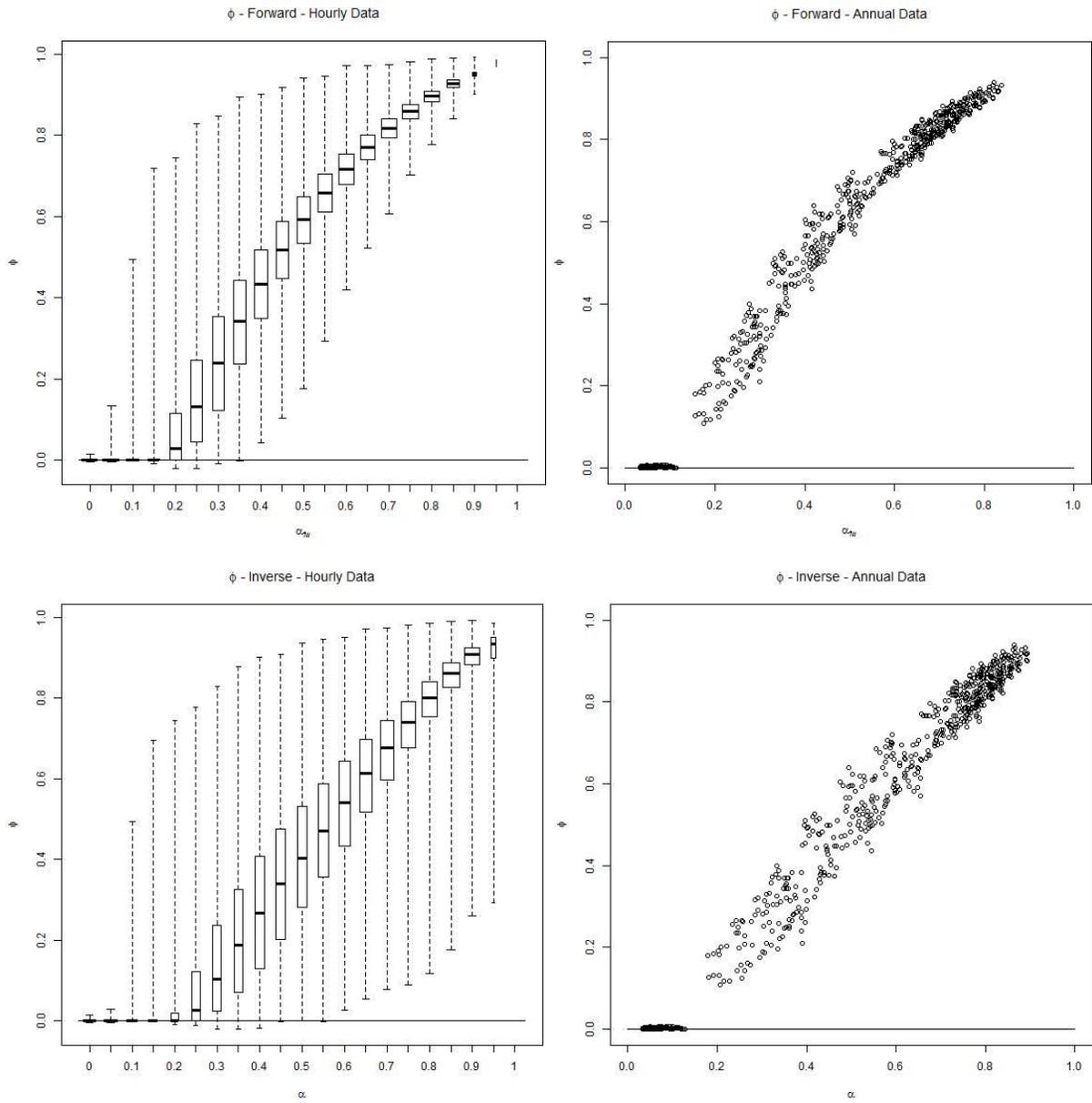


Figure 1: Sub-additivity coefficient ( $\Phi$ ) from the REGCAP simulation

A way to approximate this key parameter  $\Phi$  that reproduces this low infiltration behavior better is to use the exponential form. The use of this function makes it very complicated to have equivalent forward and inverse models. As a result it yields two different models respecting the same physical limits and the same trend:

- the **exponential forward sub-additivity (EFSA)**:

$$Q_t = Q_f + \exp\left(-k_{fw} \frac{Q_f}{Q_{inf}}\right) Q_{inf} \quad (15)$$

- the **exponential inverse sub-additivity (EISA)**:

$$Q_f = Q_t - \exp\left(-k_{inv} \left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf} \quad (16)$$

As explained in Appendix D, we optimized the coefficients  $k_{fw}$  and  $k_{inv}$  to best approximate the simulation results. We found different values for the annual and hourly data, as presented in Table 4.

For the forward model, there is no obvious reason for choosing to apply the exponential form to  $Q_{inf}$  rather than  $Q_f$ . Another model is therefore tested:

$$Q_t = \exp\left(-k'_{fw} \frac{Q_{inf}}{Q_f}\right) Q_f + Q_{inf} \quad (17)$$

If  $k'_{fw}$  equals to 1, it is the Levins' model suggested in the early eighties and found to give very similar results than the quadrature model. But for this study, the coefficient is once again optimized to best approximate the simulation results (cf table 4). For this reason, this model is referred here as the **modified Levins sub-additivity (MLSA)**.

## 4. RESULTS

### Simulation results

The REGCAP simulations generate the values of  $Q_t$ ,  $Q_f$  and  $Q_{inf}$  over a wide range of conditions. The results are presented in Figure 2; demonstrating the range and changes in airflow in air changes per hour with varying infiltration fractions ( $\alpha$ ).

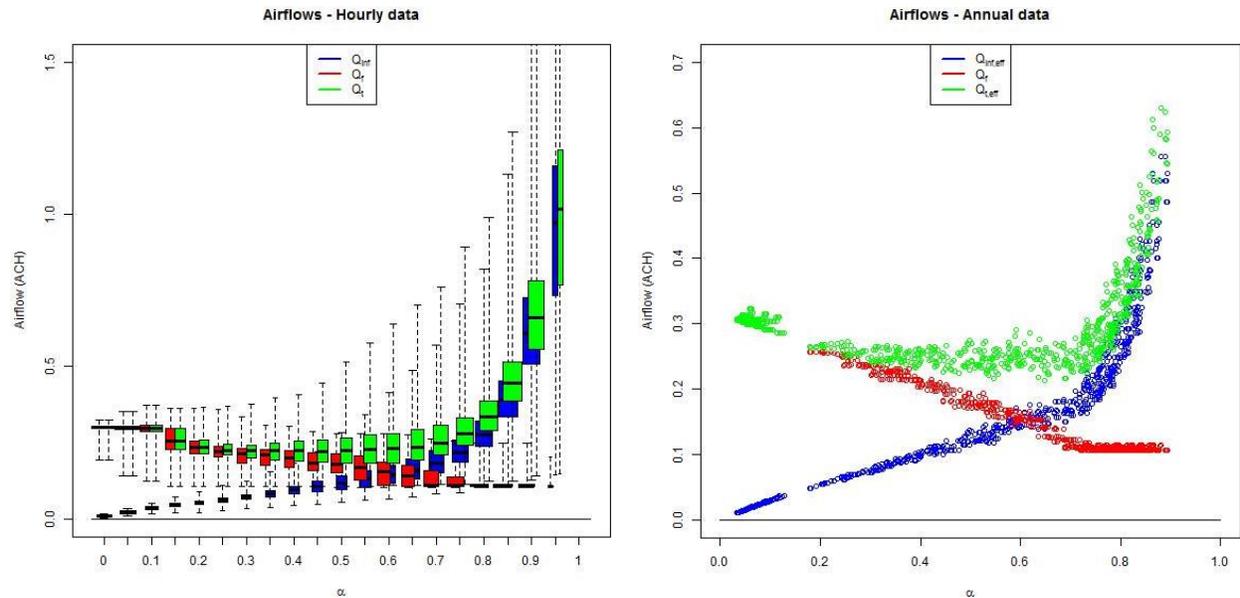


Figure 2: Infiltration, fan and total ventilation flows from the REGCAP simulation

For the hourly data, the high number of points (over six million) requires the use of summary statistics, represented by box-and-whisker plots. We sorted the data into 20 bins by infiltration fraction and each bin is represented by a box. The box widths are proportional to the square-root of the number of observations in the bin. The bottom and top of the box are the first and third quartiles, and the black band inside is the median. The ends of the whiskers represent the minimum and maximum of the data. On average, a box represents more than 300,000 points, which explains the high variance. When several parameters are plotted, each of them is identified by a color and the horizontal offset in  $\alpha$  between them is only for the sake of clarity.

For the annual analysis, there is a single result for each of the parameter combinations in Table 1. This reduced number of points (720) allows all the individual results to be shown. Compared to the hourly data, there are less extreme values and no point with  $\alpha$  above 0.9. We can observe a gap around  $\alpha = 0.15$ , which can be explained by the lack of an intermediate value between 0.6 ACH and 3 ACH in the airtightness levels of the simulated houses.

A detailed parameter study is presented in Appendix E, with the impact of each input value on the sub-additivity coefficient.

## Simplified models errors

The models are evaluated by comparing the air flow prediction to the one obtained with the simulation. The forward model aims at predicting the total airflow so the error is given by:

$$E_{fw} = \frac{Q_{t,model} - Q_{t,sim}}{Q_{t,sim}} \quad (18)$$

In the same way, the error for the inverse model is the difference between the predicted and simulated fan flows. It is still divided by the total airflow since a division by a fan flow close to zero would give a significant error but the impact on the total ventilation would be very small.

$$E_{inv} = \frac{Q_{f,model} - Q_{f,sim}}{Q_{t,sim}} \quad (19)$$

As shown in Figure 3 and 4, an over-prediction of the total ventilation ( $E_{fw} > 0$ ) results in an under-prediction of the fan flow with the equivalent inverse model ( $E_{inv} < 0$ ). We do not observe this for the exponential model since the forward and inverse models are not equivalent. For the additivity model, the two errors have simply opposing values, but there is no such symmetry for the other models. The inverse error for the quadrature model reaches higher values than the forward error for high infiltration fractions. In the same way, the half-fan model gives a higher peak in the inverse error than the forward one.

The errors for the annual data are displayed in Figure 4. The trends are similar to those of the hourly errors. However since they are effective values,  $Q_t$  and  $Q_{inf}$  are smaller than a simple annual average, unlike  $Q_f$  which is constant over the year. As a result we observe smaller over-predictions but greater under-predictions for the forward models, and the opposite for the inverse models.

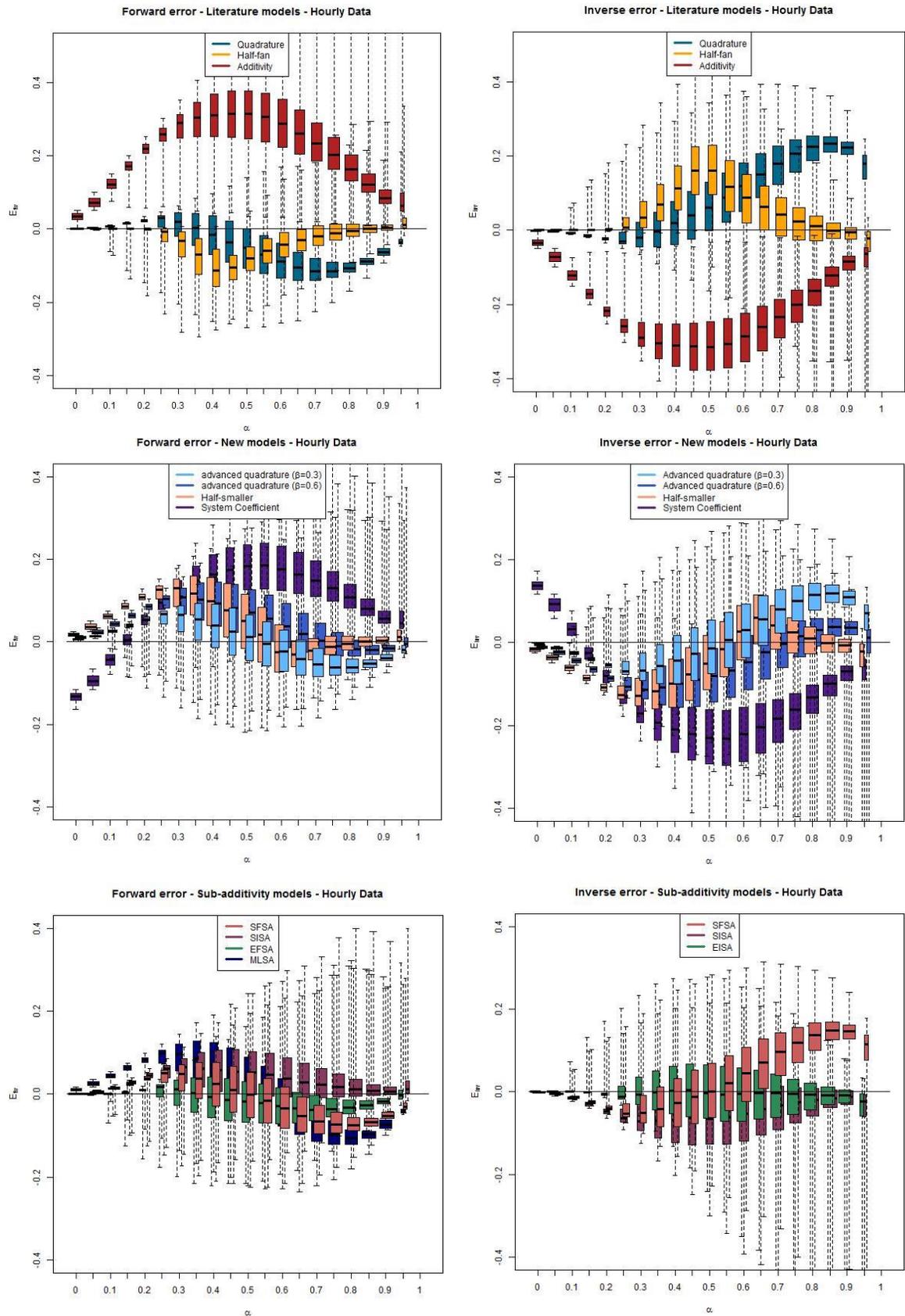


Figure 3: Forward and Inverse errors of the superposition models compared with the hourly simulations

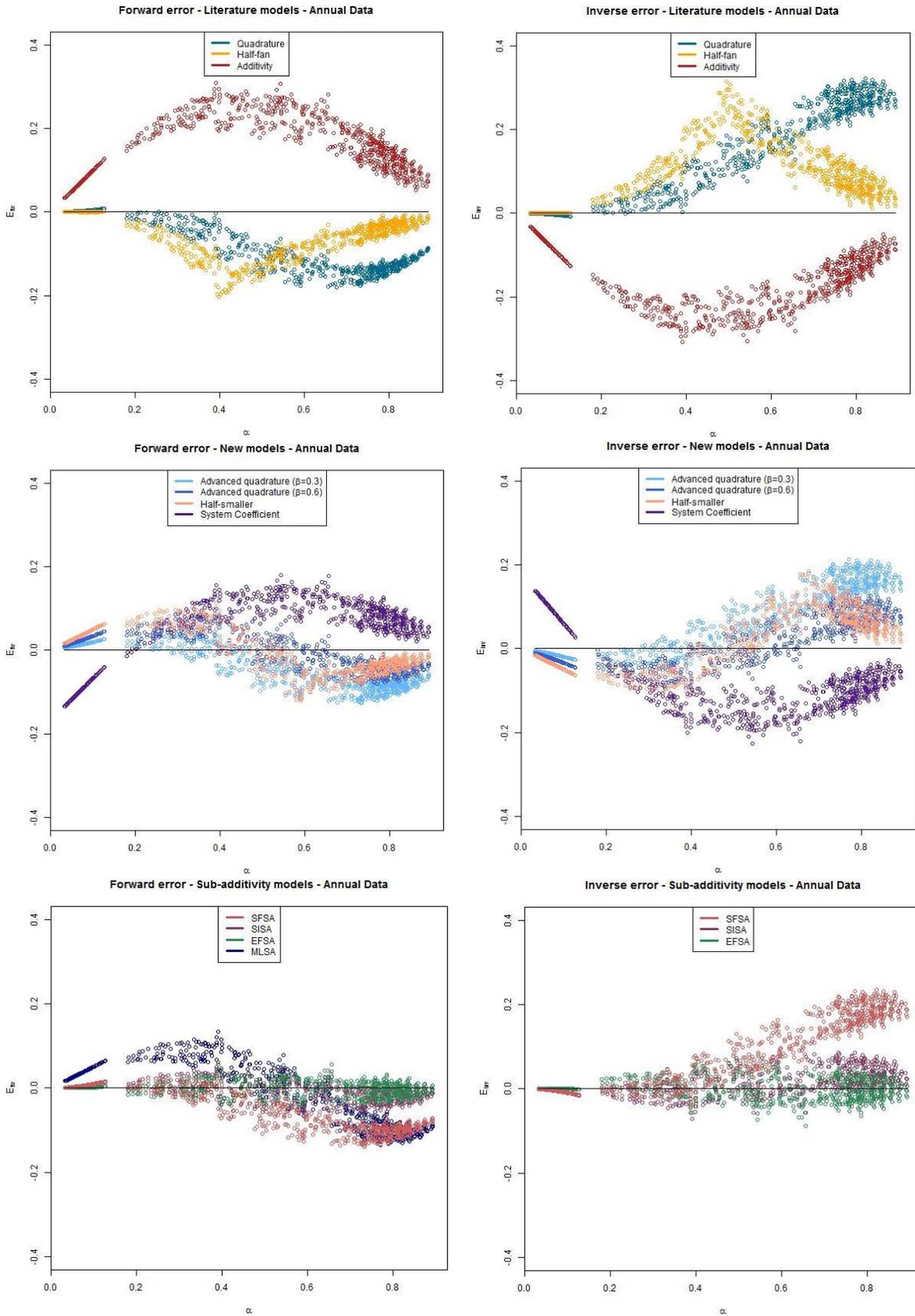


Figure 4: Forward and Inverse errors of the superposition models compared with the annual simulations

**Table 5: Error on the model predictions for the hourly data**

Model	Forward error				Inverse error				
	Bias	RMS	Max. Med.	Max. 90%	Bias	RMS	Max. Med.	Max. 90%	
Additivity	20.9%	21.8%	31.5%	34.1%	-20.9%	21.8%	31.5%	34.1%	
Simple quadrature	-3.95%	6.65%	11.6%	13.1%	7.99%	11.0%	23.3%	25.2%	
Advanced quadrature	$\beta=0.3$	0.25%	5.76%	6.73%	9.82%	1.19%	8.01%	11.9%	15.2%
	$\beta=0.6$	4.24%	6.95%	10.8%	11.4%	-4.36%	8.27%	11.3%	13.4%
Half-fan	-2.85%	4.78%	11.3%	9.22%	4.20%	7.65%	16.1%	15.8%	
Half-smaller	4.32%	7.37%	12.9%	12.0%	-4.13%	8.63%	12.9%	14.1%	
Constant system coefficient	9.07%	12.8%	18.5%	21.5%	-12.1%	15.8%	23.2%	26.7%	
SFSA	-1.03%	5.60%	7.57%	10.3%	3.01%	8.32%	14.9%	17.1%	
SISA	3.29%	5.39%	6.57%	9.43%	-4.12%	7.02%	7.21%	12.7%	
EFSA	-1.15%	4.01%	3.78%	8.03%			-		
EISA			-		-0.61%	5.57%	2.31%	11.3%	
MLSA	0.51%	7.93%	10.7%	12.9%			-		

**Table 6: Error on the model predictions for the annual data**

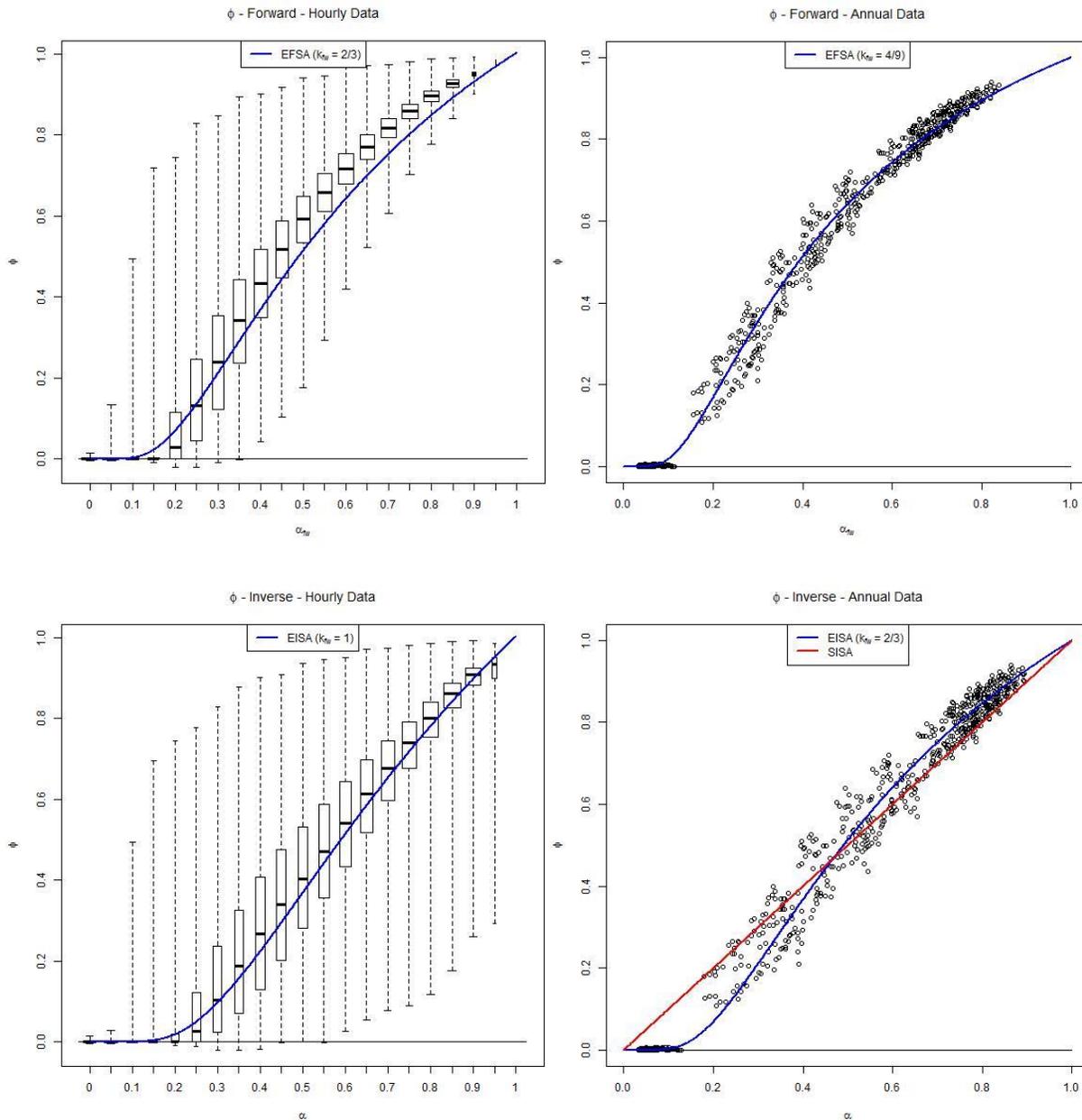
Model	Forward error			Inverse error			
	Bias	RMS	Max.	Bias	RMS	Max.	
Additivity	17.4%	17.5%	30.9%	-17.4%	17.5%	30.9%	
Quadrature	-7.51%	7.82%	18.1%	11.72%	12.1%	32.2%	
Advanced quadrature	$\beta=0.3$	-3.29%	4.88%	12.7%	5.40%	7.10%	21.2%
	$\beta=0.6$	0.72%	3.99%	10.0%	-0.05%	4.94%	12.7%
Half-fan	-6.43%	6.58%	20.3%	9.86%	10.1%	31.6%	
Half-smaller	0.62%	5.25%	12.2%	0.65%	6.58%	18.6%	
Constant system coefficient	5.33%	8.67%	17.9%	-7.69%	10.9%	22.6%	
SFSA	-4.53%	5.22%	13.9%	6.96%	7.73%	23.4%	
SISA	0.32%	1.95%	6.18%	0.68%	2.60%	8.65%	
EFSA	-0.15%	1.57%	5.55%				
EISA				0.18%	2.22%	8.85%	
MLSA	0.85%	6.55%	13.4%				

## Discussion

The characteristics of the hourly and annual errors are presented respectively in Tables 5 and 6. The various simulations give results covering a wide range of infiltration ratio ( $\alpha$ ) but not evenly dispersed, with for example fewer points around  $\alpha=0.15$ . In order to compensate for this disparity, we calculated the bias and RMS of the errors for 20 bins of  $\alpha$  values, and then we equally weighted them. The bias is the error over the full range of house and weather parameters exercised in this study. The RMS is representative of the error for an individual home and is most useful for most applications – such as sizing fans for an individual home to meet a ventilation standard, such as ASHRAE 62.2. Because of the high number of points, the maximum error is not meaningful for the hourly error. We use instead the maximum median among the 20 groups of data, and the maximum of 90% of the data. The best models for each case are presented in Appendix F.

Each of the models have the same physical limits with  $Q_t$  equals to  $Q_f$  when  $\alpha$  tends towards 0 (no infiltration) and  $Q_t$  equals to  $Q_{inf}$  when  $\alpha$  tends towards 1 (no mechanical ventilation). That is the reason why the errors tend to 0 at the extreme values of  $\alpha$ . One can notice that some models, such as additivity and half-fan, have their maximum errors for  $\alpha$  close to 0.5 whereas the quadrature model has its maximum error around 0.7 for the forward calculation and 0.8 for the inverse one. This means that depending on the airtightness of the building, the ranking of the best models is different, and this could be one of the reasons why the previous studies did not agree on which model to recommend.

For the four cases (hourly vs annual and forward vs. backward) the exponential models always give the best predictions, with biases around or below 1%, RMS ranging from 1.5% to 5.5% and maximums around or under 10%. As we can see from Figure 5, the approximation of the sub-additivity coefficient is good at capturing the relationship between  $\Phi$  and  $\alpha$ .



**Figure 5: Sub-additivity coefficient ( $\Phi$ ) from the REGCAP simulation with the best model(s) for each case**

The simple additivity model has the largest errors, with bias and RMS errors of around 20% and maximums above 30%. It consistently overpredicts  $Q_t$  (and therefore underpredicts  $Q_f$ ), and the error has a parabolic shape with a maximum for an infiltration fraction  $\alpha$  close to 0.4.

The quadrature model gives good predictions for tight houses ( $\alpha < 0.4$ ), but it underpredicts  $Q_t$  for leakier houses. The maximum error is found in the vicinity of  $\alpha = 0.8$  and is almost doubled from the forward to the inverse model, reaching 32% for the annual data.

The use of advanced quadrature coefficients enables to have biases close to zero. These hourly and annual models overpredict  $Q_t$  for tight houses and underpredict for leaky houses with maximum errors of around 10% for  $\alpha=0.25$  and  $\alpha=0.75$ . The RMS errors around 5% are also smaller compared to the simple quadrature, and these two models are therefore a good improvement.

The half-fan model from the literature turns out to be good for extreme values of  $\alpha$ , but shows a peak error around  $\alpha=0.4$ , with a maximum error reaching 32% for the annual inverse case. The half-smaller model in comparison is better for the annual data, with errors very similar to the advanced quadrature, but is no improvement for the hourly data.

The constant system coefficient model is overall a slight improvement compared to simple additivity, but the biases and RMS errors are still significant, especially for the hourly data.

The error obtained with the SFSA model are very close to the advanced quadrature ones. The hourly cases have biases under 3% and RMS around 5% for the forward case and 8% for the inverse one. When applied to the annual data, the RMS is still as good, but the biases reach 5% and the maximum errors 20%.

The SISA model also gives very good results, and it is almost as good as the exponential for the annual data with biases below 1%, RMS around 2% and maximums at 6% and 8.7%. As shown in Figure 5, the approximation of the sub-additivity coefficient is not as good as the exponential one for very low  $\alpha$ . However for this range the coefficient applies to very small values of  $Q_{inf}$  and as a result does not result in large errors in the  $Q_t$  or  $Q_f$  predictions. There is no reason to prefer this model to the exponential one for the forward prediction, but it has a simpler expression for the inverse prediction. It also has the advantage of not having  $Q_t$  as a denominator, which, unlike  $Q_{inf}$ , can never equal to zero and may therefore be a good option for calculations determining fan size requirements to meet total ventilation rates.

## 5. CONCLUSIONS

The consensus superposition model has a bias in its predictions on the order of 5% and an RMS error of approximately 10% for unbalanced systems. We would expect a 10% error for an individual home to result in measurable impacts on energy consumption or IAQ. ASHRAE Standard 62.2's use of simple additivity is significantly worse, with a 20% bias and a 20% RMS error.

We have explored a variety of other models and found that several of them are superior to simple quadrature in various ways. The model that is superior to all others is the exponential model. It takes different forms for the forward and inverse calculations, and it has different optimized coefficients for the hourly and annual forms:

**Table 7: Exponential sub-additivity model**

	Forward	Inverse
<b>Hourly</b>	$Q_t = Q_f + \exp\left(-\frac{2}{3}\frac{Q_f}{Q_{inf}}\right) Q_{inf}$	$Q_f = Q_t - \exp\left(-\left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf}$
<b>Annual</b>	$Q_t = Q_f + \exp\left(-\frac{4}{9}\frac{Q_f}{Q_{inf}}\right) Q_{inf}$	$Q_f = Q_t - \exp\left(-\frac{2}{3}\left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf}$

While this model set is superior an even simpler application may be desired in some cases. When using annual data the following simple expression is almost as good as its exponential counterpart:

$$Q_f = Q_t - \frac{Q_{inf}^2}{Q_t} \quad (20)$$

For hourly data there is an advanced quadrature form that works reasonably well:

$$Q_t = \sqrt{Q_f^2 + Q_{inf}^2 + 0.3Q_f Q_{inf}} \quad (21)$$

A linear expression (in the forward direction) that works almost as well for hourly data may also be useful:

$$Q_t = Q_f + \frac{Q_{inf}^2}{Q_f + Q_{inf}} \quad (22)$$

These last three models have accuracy in both forward and backward directions, but their expressions become complicated quadratics when inverting from the way they are shown. Thus it may not be much more complicated to use the more accurate exponential forms.



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## Appendix A: Use of the REGCAP data

- Collection of the data from the REGCAP output files

The results of each REGCAP simulation are written in 4 output files, including:

- .rco output file: contains 42 outputs for every minute, including temperatures, air flows, energy use and IAQ.
- .rc2 output file: summarizes the annual simulation with averages of 24 outputs, including temperatures, air flows, energy use and IAQ.

For the purpose of this study, we wrote R programs to collect the air flows ( $Q_{inf}$ ,  $Q_f$  and  $Q_t$ ) of each simulation. The exact locations of these parameters for the hourly and annual studies are given respectively in tables 8 and 9.

**Table 8: Location of the parameters collected for the hourly study**

Airflow	Simulation		Output file	Output name
	No fan	With fan		
$Q_{inf}$	x		.rco	ACH (col.U)
$Q_t$		x		
$Q_{f0}$		x		ventSum (col. W)

**Table 9: Location of the parameters collected for the annual study**

Airflow	Formula	Parameters	Simulation		Output file	Output name
			No fan	With fan		
$Q_{inf,eff}$	$= \frac{AEQ}{MRE}$	AEQ: ASHRAE target ventilation rate (ACH)		x	.rc2	Aeq (col. S)
		MRE: Mean relative exposure	x			meanRelExpReal (col. L)
$Q_{t,eff}$	$= \frac{AEQ}{MRE'}$	AEQ: ASHRAE target ventilation rate (ACH)		x		Aeq (col. S)
		MRE': Mean relative exposure		x		meanRelExpReal (col. L)
$Q_{f0}$	-	-		x	.rco	ventSum (col. W)

Notes:

- The parameters from .rco files are hourly averaged before being collected
- $Q_f$  is constant over a year
- For the annual study, the effective values of  $Q_{inf}$  and  $Q_t$  are calculated
- The AEQ value is read from the output file of the simulation with mechanical ventilation but it would not make any difference to read it from the simulation without fan.
- The exponent in the pressure law is constant for the simulation ( $n=0.67$ ) but in reality it won't be the same for very tight houses (up to 0.8) or very leaky ones (down to 0.55). This can question the validity of the models for these cases. However for these extreme airtightness levels, there is one dominant flow that makes the superposition issue less important.
- Fan flows are corrected to account for the fact that REGCAP uses the density of internal air for its calculations but the supply fan will be entered as the volumetric flow at outside conditions. The REGCAP calculations are made with mass flows. The conversion to fan flow uses the indoor density for exhaust fans and outdoor density for supply fans. On the other hand,  $Q_t$  and  $Q_{inf}$  are using indoor air density for reference. As a consequence a mass balance between  $Q_t$  and  $Q_f$  can result in different volumetric air flows. A correction is therefore applied to the supply fan flows.

For the hourly data:

$$Q_f = \begin{cases} \frac{T_{in}}{T_{out}} Q_{f0} & \text{for supply fans} \\ Q_{f0} & \text{for exhaust fans} \end{cases} \quad (19)$$

With  $T_{in}$  and  $T_{out}$  the hourly indoor and outdoor temperatures.

For the annual data:

$$Q_f = \begin{cases} \frac{\langle T_{in} \rangle}{\langle T_{out} \rangle} Q_{f0} & \text{for supply fans} \\ Q_{f0} & \text{for exhaust fans} \end{cases} \quad (20)$$

With  $\langle T_{in} \rangle$  and  $\langle T_{out} \rangle$  the annual averages of the indoor and outdoor temperatures.

## Appendix B: Optimization of the advanced quadrature model

The advanced quadrature is similar to the quadrature model but with an additional correction term driven by the coefficient  $\beta$ :

$$Q_t = \sqrt{Q_f^2 + Q_{inf}^2 + \beta Q_f Q_{inf}} \quad (21)$$

If  $\beta=2$  this is the additivity model that will always overpredict  $Q_t$ . If  $\beta=0$  this is the simple quadrature model that tends to underpredict  $Q_t$ . So by taking intermediate values of  $\beta$  we can get better predictions of  $Q_t$ .

For the forward model, the ideal advanced quadrature coefficient  $\beta$  verifying (21) is defined as follows:

$$\beta = \frac{Q_t^2 - Q_f^2 - Q_{inf}^2}{Q_f \cdot Q_{inf}} \quad (22)$$

As presented in Figure 6, we used the REGCAP simulation results to plot this coefficient for the hourly and annual data. The equally weighted averages are  $\bar{\beta} = 0.271$  for the hourly data and  $\bar{\beta} = 0.461$  for the annual one, but we can observe that the value depends highly on the infiltration fraction.

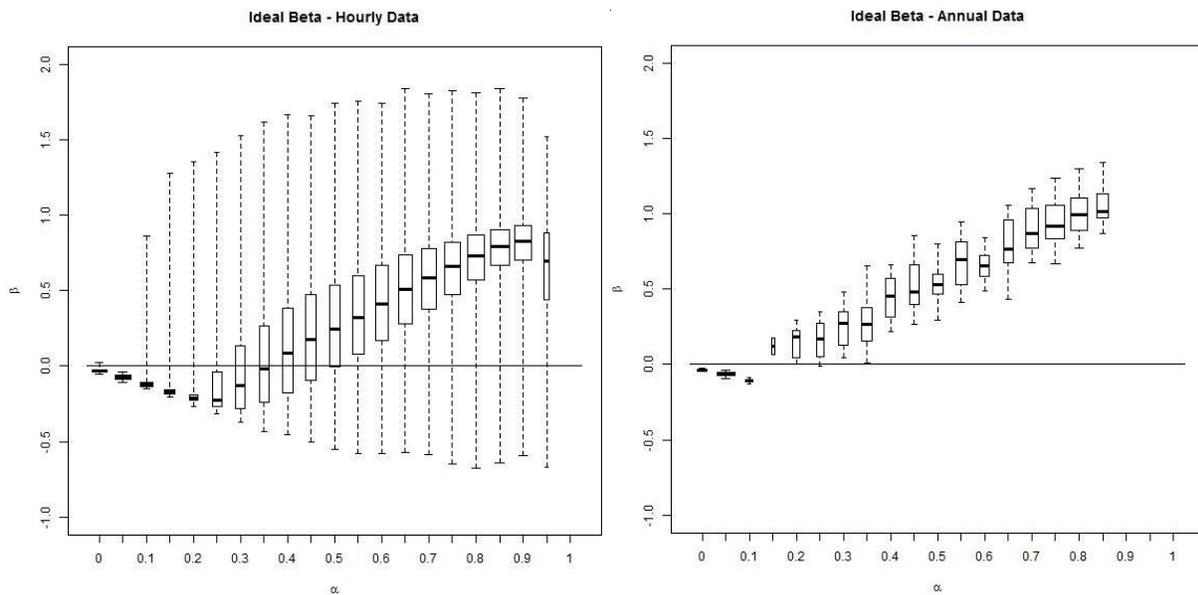
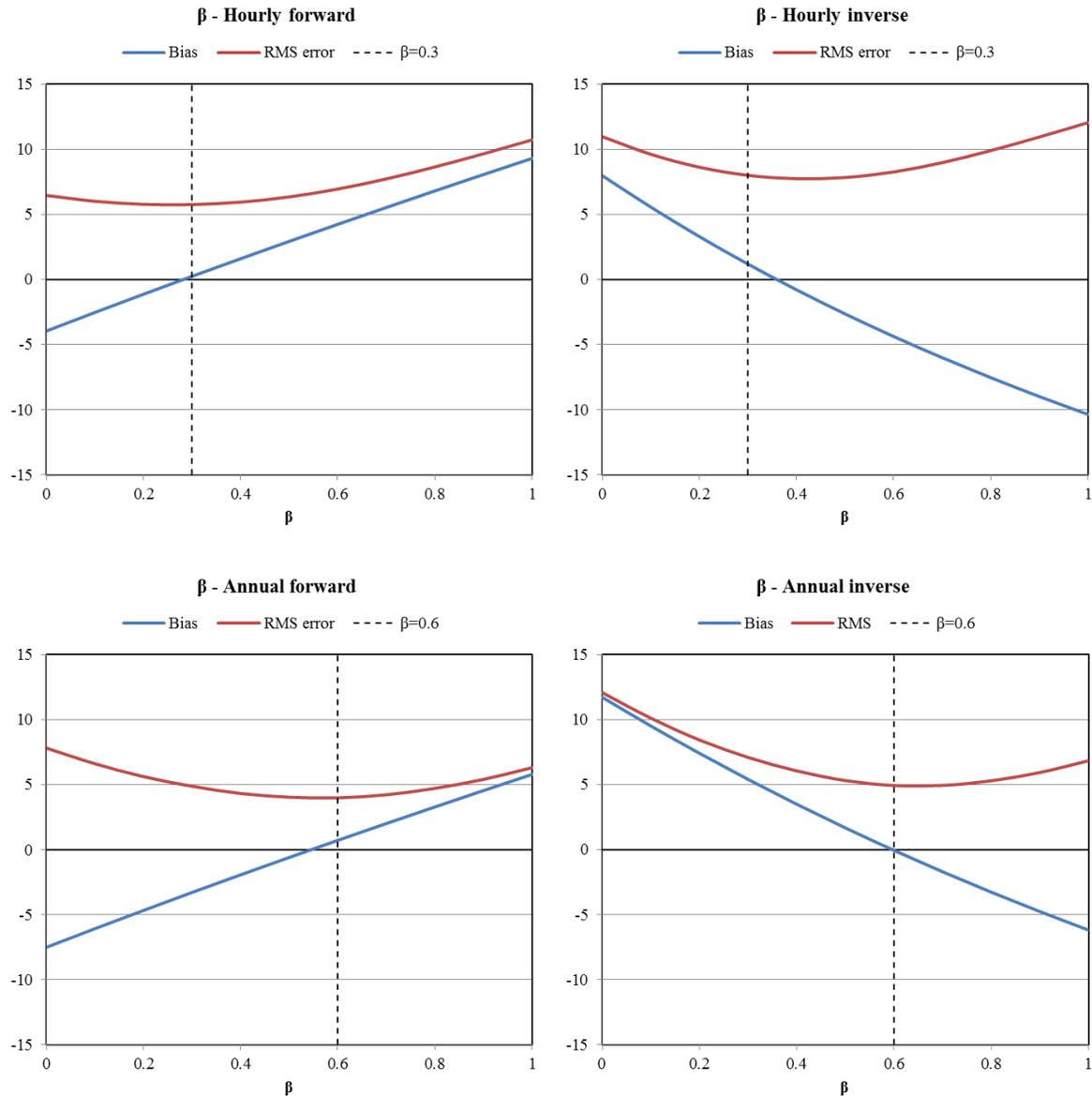


Figure 6: Hourly and annual ideal advanced quadrature coefficient ( $\beta$ ) obtained with REGCAP

These averages give an idea of the best values to use, but a more detailed analysis of the biases and RMS errors presented in Figure 7 enables to select slightly better coefficients.

For the hourly data,  $\beta=0.3$  minimizes both the bias and RMS error for the forward model, and is also pretty close to the optimum point for the inverse one. Concerning the annual data,  $\beta=0.6$  minimizes both the bias and RMS error for the inverse model, and is pretty close to the optimum point for the forward one.



**Figure 7: Impact of the advanced quadrature coefficient  $\beta$  on the bias and RMS error for the forward and inverse models compared with the hourly and annual REGCAP data**

## Appendix C: System coefficient model

The system coefficient is a parameter that could be used for a future 62.2 ASHRAE Standard, which means for annual inverse calculations. The infiltration flow ( $Q_{inf}$ ) and targeted total ventilation flow ( $Q_t$ ) are known, and the fan has to be sized accordingly. This system coefficient is defined as follows:

$$D = \frac{Q_f}{Q_t - Q_{inf}} \quad (23)$$

As a result:

$$\begin{cases} Q_f = D(Q_t - Q_{inf}) \\ Q_t = \frac{1}{D} Q_f + Q_{inf} \end{cases} \quad (24)$$

The current 62.2 ASHRAE Standard uses the additivity model which corresponds to a system coefficient equal to 1, but this model is always overpredicting the total ventilation flow (i.e. undersizing the fan).

A proposal is currently studied to modify this system coefficient. The actual proposal is to reduce the fan flow by 15% for balanced systems and leave unbalanced systems unchanged. Since from first principles balanced systems are unaffected by infiltration, this approach makes no physical sense. The closest physical model to this is to reduce the total and then increase the fan flow for unbalanced systems—leaving balanced systems unchanged. The fact that the total rate is reduced by this method is a separate issue, but not one addressed herein. Rather we simply consider the value of the model assuming the same target effective ventilation in both cases. For unbalanced mechanical ventilation, the suggested value is:  $\frac{1}{D} = 0.85$ .

The figure 8 displays the system coefficient from the REGCAP simulation and compares it to the value from the proposal. It seems that a constant value is inappropriate to fit the data, and the one suggested would be good only for a small range of airtightness levels around  $\alpha=0.2$ .

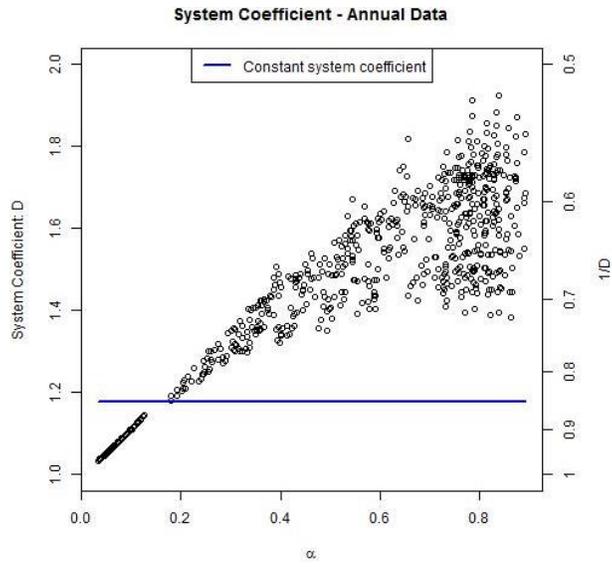


Figure 8: System coefficient calculated with REGCAP compared with the proposal

## Appendix D: Optimization of the exponential models

We suggested three exponential superposition models:

- the exponential forward sub-additivity (EFSA):  $Q_t = Q_f + \exp\left(-k_{fw} \frac{Q_f}{Q_{inf}}\right) Q_{inf}$
- the exponential inverse sub-additivity (EISA):  $Q_f = Q_t - \exp\left(-k_{inv} \left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf}$
- the modified Levins sub-additivity (MLSA):  $Q_t = \exp\left(-k'_{fw} \frac{Q_{inf}}{Q_f}\right) Q_f + Q_{inf}$

The exponential coefficients  $k_{fw}$ ,  $k_{inv}$  and  $k'_{fw}$  are optimized to best approximate the simulation results. We have calculated for each model, and for both hourly and annual data, the bias and RMS error induced as a function of the exponential coefficient. The results are presented in Figure 9. As shown in Table 10 the minimum of the bias and RMS errors are obtained for the same or very close exponential coefficient values. The only exception is the hourly MLSA model, but the RMS error for the coefficient minimizing the bias is very close to the lowest one.

We chose to express the exponential coefficients as fractions. The only model for which it could be critical is the hourly EFSA with a selected coefficient of  $\frac{2}{3}$  instead of 0.6. However, the RMS error are almost the same and the bias induced by the fraction is only 1%, which is considered small enough given the uncertainties due to the use of a simulation tool.

**Table 10: Optimized and selected values of the exponential coefficient for each model**

Model	Bias minimization	RMS error minimization	Selected coefficient
Hourly EFSA	$k_{fw} = 0.60$	$k_{fw} = 0.60$	$k_{fw} = \frac{2}{3} \approx 0.67$
Annual EFSA	$k_{fw} = 0.44$	$k_{fw} = 0.44$	$k_{fw} = \frac{4}{9} \approx 0.44$
Hourly EISA	$k_{inv} = 1.06$	$k_{inv} = 1.04$	$k_{inv} = 1$
Annual EISA	$k_{inv} = 0.66$	$k_{inv} = 0.67$	$k_{inv} = \frac{2}{3} \approx 0.67$
Hourly MLSA	$k'_{fw} = 0.69$	$k'_{fw} = 0.90$	$k'_{fw} = \frac{2}{3} \approx 0.67$
Annual MLSA	$k'_{fw} = 0.54$	$k'_{fw} = 0.58$	$k'_{fw} = 0.5$

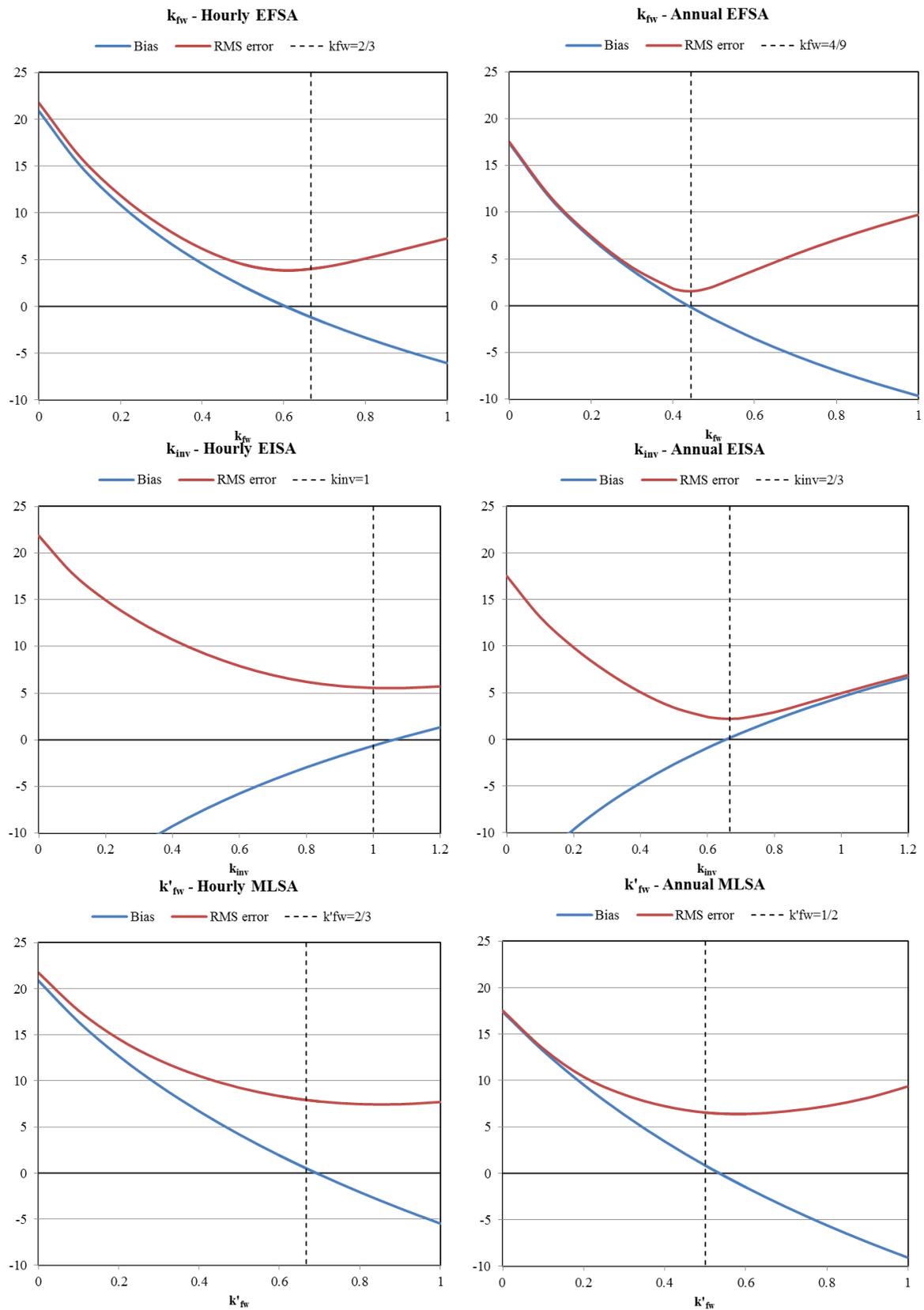


Figure 9: Impact of the exponential coefficient on the bias and RMS error for the forward and inverse models compared with the hourly and annual REGCAP data

## Appendix E: Impact of the simulation parameters on the sub-additivity coefficient (annual – forward case)

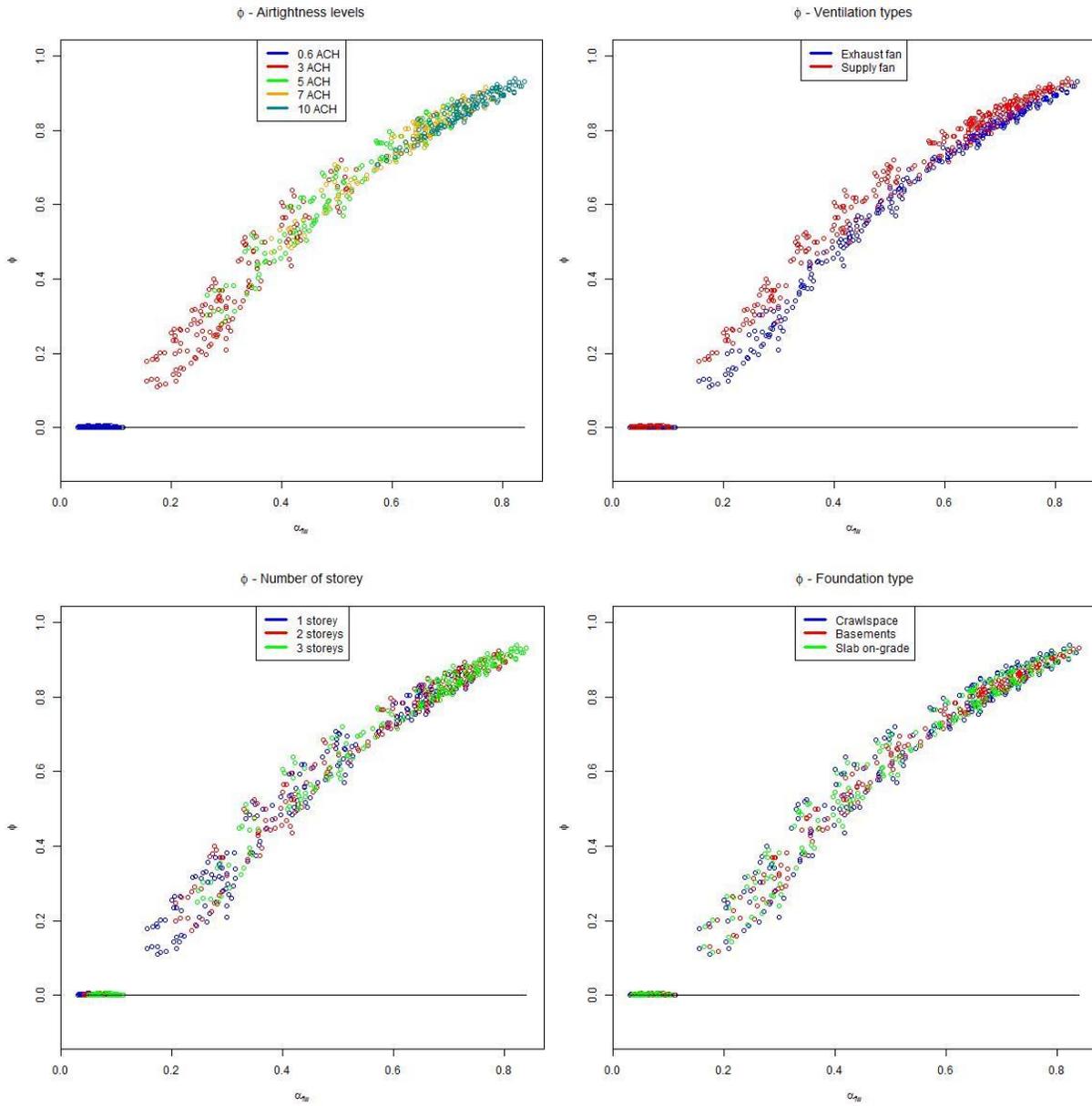
In order to better understand the REGCAP simulation results, we have studied the impact of each input parameter on the sub-additivity coefficient. For this purpose, we used the annual data since it is much easier to display than the hourly one. Moreover, we used the forward case, which means that the coefficient is plotted against  $\alpha_{fw} = Q_{inf}/(Q_t - Q_{inf})$ , but the inverse case would give the same conclusions.

The first plot shows the sub-additivity coefficient according to the airtightness level. The very tight houses with 0.6 ACH at 50Pa have a sub-additivity coefficient equal to zero: the total ventilation flow is the fan flow. This means that the fan induces a pressure difference between both sides of the envelope that is always bigger than the local dP induced by the stack and wind effect. As a result, for exhaust ventilation there will be no exfiltration through the envelope, and for supply ventilation there will be no infiltration. When the airtightness level decreases, a smaller fan flow is required. From 3 ACH,  $\Phi$  never equals to zero, which means that locally the wind and stack effects induce bigger pressure differences than the mechanical ventilation. This plot also confirms that the gap observed around  $\alpha=0.15$  is due to a lack of an intermediate value between 0.6 ACH and 3 ACH, and there is no reason to think that there is a discontinuity at this point.

The second plot reveals two groups of data depending on the ventilation type. Simulations with supply fans give higher values of  $\Phi$  than the one with exhaust fans. This is due to the fact that the house is normally slightly depressurized in order to balance the flows under natural infiltration. Without mechanical ventilation the internal relative pressure is therefore negative. The exhaust fan makes it go more negative whereas the supply fan makes it go more towards zero and then positive. Because of the non-linear pressure-flow relationship and the interactions with the different leak locations, this leads to different effects for supply and exhaust systems. It means that we could create two separate models depending on the ventilation type. However, this distinction is not very relevant, because the differences in the two groups are small compared to the uncertainties due to the use of a simulation tool.

The number of stories has also a visible impact. The stack effect is amplified with the height of the building, which increases the infiltrations and therefore  $\alpha$ . On the contrary, the foundation type does not seem to have a big impact. Finally, in the last plot three cities are displayed among the eight

simulated. Cold climates such as in Fairbanks, Alaska induce bigger temperature differences between the inside and outside of the house, which also increases the stack effect and therefore  $\alpha$ .



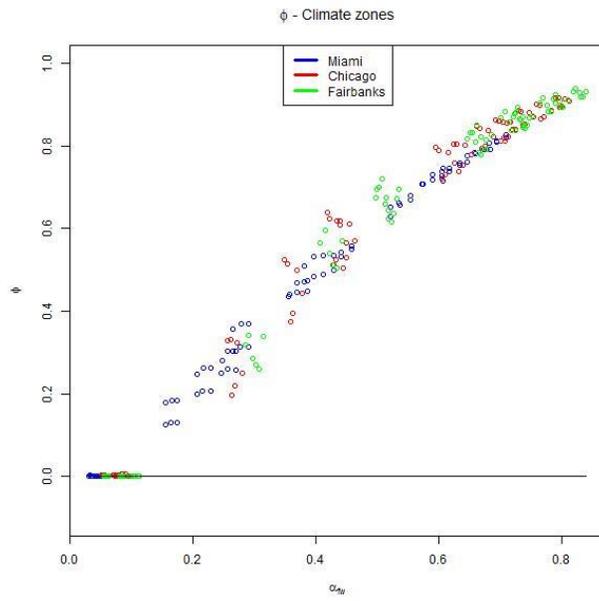


Figure 10: Impact of the input parameters on the sub-additivity coefficient ( $\Phi$ )

# Appendix F: Comparison of the best superposition models

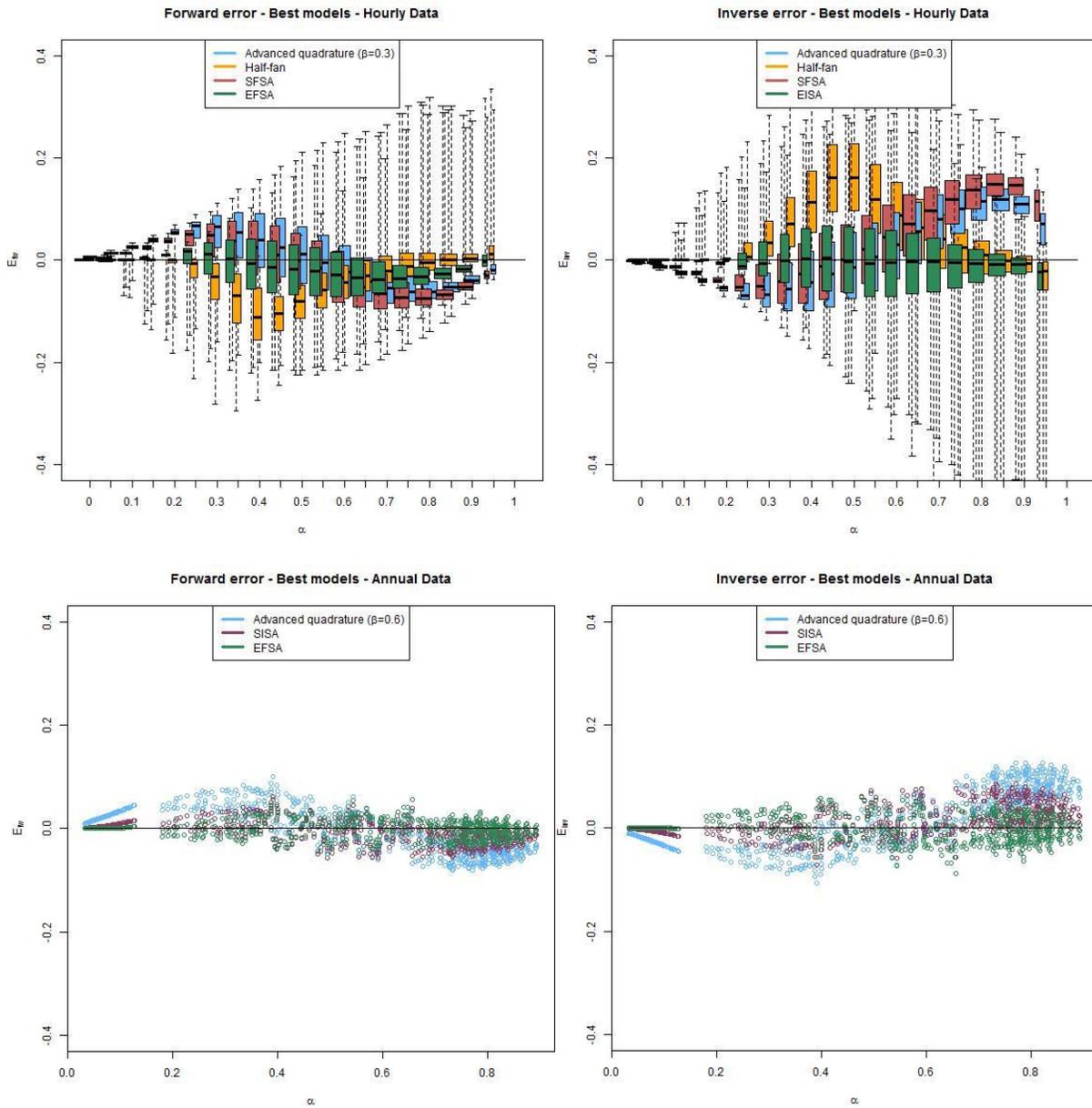


Figure 11: System coefficient calculated with REGCAP compared with the proposal